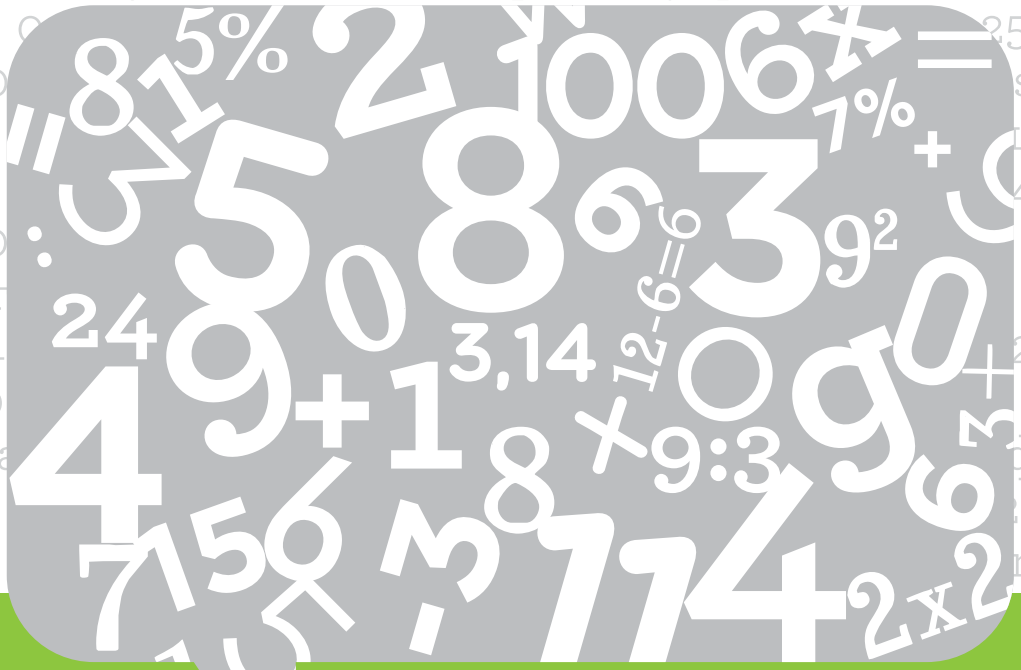

THINKING MATHEMATICS IN LOWER GRADES

REPORT
of the baseline study

unite for children

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Skopje, December 2009

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ЦЕНТАР ЗА
ГРАЃАНСКО
ОБРАЗОВАНИЕ

CONTENT

EXECUTIVE SUMMARY

The objective of the study

The baseline study for the project *Thinking Mathematics,* had an objective to *establish methodology* and *to collect baseline data,* which would then enable subsequent monitoring of project activities and measuring project impact on the quality of instruction and on the achievements of students.

More precisely:

1. To make a survey of teachers' understanding of learning and of teaching mathematics in the grade teaching cycle, related to the Ten Principles of Teaching Mathematics.
2. To make a survey of teachers' mathematics and pedagogical knowledge related to understanding the contents and the pedagogical approaches in *Thinking Mathematics.*
3. To measure students' achievements at the end of Grade 3 on issues and tasks in program areas of *Thinking Mathematics*.
4. To investigate the activities for improving teaching of mathematics at schools and the readiness to provide support to project activities in the schools involved in the project.
5. To estimate specific needs for training of teachers taking part in the project.

Conducting the study

Data were collected on a sample of 15 schools, taking part in the project *Thinking Mathematics,* and a sample of 15 schools (with similar characteristics, as a control group) that are not in the project. So, 10 teachers and 20 students from each school (at the beginning of Grade 4), i.e. a total of 299 teachers and 597 students were examined. Such an approach provides credibility in comparing the situation in project and non-project schools, at the beginning and later during the implementation of project activities. Those schools could not be considered as a representative sample of all schools in the country, and the findings could not be generalized.

Data were collected using the following instruments developed for this study:

- Scale of attitudes on mahematics and on instruction of mathematics;
- Scale of attitudes on learning and on teaching mathematics;
- Test of pedagogical knowledge of teachers (related to the Ten Principles of *Thinking Mathematics*);
- Test of mathematics knowledge for teachers;
- Questionnaire for teachers;
- Test of mathematics for students in Grade 3;
- Manual for interviews with school directors and pedagogues/psychologists.

Data processing and analysis of all examined subjects was carried out according to established indicators, and comparisons of starting baseline data in project and in non-project schools are given also.

Basic findings of the study

The indicators, a brief review of each indicator, and the main findings are given bellow.

It is concluded that, within each indicator and for each category of examinees **there is no sstatistically significant difference betwen the examinees in the project schools and those in the non-project schools.** That will make it easier to make comparisons between project and non-project schools in the subsequent monitoring of the influence of project activities.

Indicator	Brief description	Findings
Understanding of learning and of teaching mathematics	Attitudes of teachers about learning and teaching of mathematcs based on the Ten Principles.	- In general, teachers accept statements describing learning of mathematics according to understanding of learning in Thinking Mathematics and of the practice adjusted to the 10 principles in teaching mathematics. - There is an explicit awareness about the importance of applying various ways of presenting numbers, the use of life experiences and of the formative assessment in teaching. - On the other hand, the responses of teachers also indicate a considerable acceptance of teaching practice which is not adjusted to the Ten Principles, i.e. instruction focused to acquiring content rather than objectives, insufficient differentiation, too much didactics and traditional teaching style, promoting of mathematics as a discipline containing mainly a sum of notions, rules, procedures, definitions etc.
	Attitudes of teachers to mathematics and to the teaching of mathematics.	Attitudes of teachers to mathematics, as a teaching subject, and to the instruction of mathematics, are generally positive.
	Pedagogical knowledge of teachers related to the approaches promoted by the Project.	Results of teachers at the test on pedagogical knowledge are low. The average percentage of the correct answers at the test is 35% in the project schools, and 31% in the non-project ones.
	Teachers expectations from their students, related to achievements in mathematics.	- - Teachers' expectations relating to the period when the majority of students would be able to do particular tasks, are lower than those that could be achieved by students, provided that the instruction would use the approaches developed by the Project. However, in comparison with the curriculum requirements, teachers consider that students could not achieve more than that prescribed by curricula. - Teachers consider that the most important outcomes of learning mathematics at the end of the first cycle, are the technical skills and the accuracy in mathematics, especially in doing the tasks related to addition, subtraction, multiplication and division, but the creativity is considered as less important. - About 40% of teachers consider that they do not have sufficient freedom to select and use a methods' approach in the teaching of mathematics that they think is the most adequate one.
	Familiarity with mathematics curriculum for the subsequent education cycles.	Teachers, in project as well as in non-project schools, are not familiar to a sufficient extent with mathematics curriculum for the subsequent education cycles of primary education, and in particular, of the last one (Grades 7 – 9).

Indicator	Brief description	Findings
Teachers' knowledge of mathematics	The knowledge of teachers and their understanding of the concepts of number, operations and characteristics of operations, doing test tasks and problems' solving.	- The level of mathematics knowledge of teachers, measured by test items, is low. Namely, the average percentage of the correct answers on the test is 34% in the project, and 33% in the non-project schools, although the tasks used to measure this knowledge of teachers did not exceed the curriculum requirements for the second cycle in primary education. - The percentage of correct answers on the tasks given in the test as a whole and in the three areas covered by Project program ranges between 30%, on tasks in the test of operations and characteristics of operations (contents to which, according to curriculum, is devoted most of the time for their elaboration with children), and up to 36% on textual tasks and problem solving situations.
Support to changes in the teaching of mathematics	Satisfaction with students' achievements.	The most often practiced approaches in improving the teaching in the project schools were those of the complementary training of teachers and their mutual cooperation. Majority of teachers consider that they cooperate well with colleagues and could rely on support at school when introducing innovations in the teaching of mathematics, however, the cooperation between the grade teachers and the subject teachers of mathematics is insufficient.
	Support in improving the teaching of mathematics.	The most often practiced approaches in improving the teaching in the project schools were those of the complementary training of teachers and their mutual cooperation. Majority of teachers consider that they cooperate well with colleagues and could rely on support at school when introducing innovations in the teaching of mathematics, however, the cooperation between the grade teachers and the subject teachers of mathematics is insufficient
	Equipment of schools for the instruction of mathematics.	The equipment of classrooms with manipulative aids is not satisfactory, and at the same time, the management staff lacks professional knowledge of what is needed for the teaching of mathematics in the grade teaching cycle, and it is not informed sufficiently about its school conditions and needs.
	Readiness of the management staff to give support to implementing new approaches.	Although the management staff, before starting the project activities in school, is not fully informed concerning the project objectives and activities, there is an openness to give support to new projects and approaches to teaching (including UNICEF project on *Thinking Mathematics*).

Indicator	Brief description	Findings
Achievement of students	Achievement of students at the test in mathematics which contains tasks that measure conceptual and procedural knowledge, understanding and applying of natural numbers, the four operations and their characteristics, as well as doing textual tasks and problems' solving.	Achievements of students on the mathematics test are low, compared to the expected results at the end of Grade 3, which are prescribed by the curriculum. The average score of students is 1/3 out of the maximum possible one. The lowest percentage of the correct answers test in project schools is 33% on the tasks related to Operations and characteristics of operations. This is to a large extent, a worrying state taking into consideration the fact that most of the time in teaching mathematics is devoted to the four mathematics operations and to their characteristics.

Recommendations

Taking into considerations the findings presented above, it is expected that the coherent and quality implementation of the project program *Thinking Mathematics*, followed by continuing support to teachers, would raise the level of teachers' pedagogical and mathematics knowledge and students' achievements in project schools. In addition to the thoughtful and well-developed training program it would be useful to undertake the following activities:

With management teams in schools

- UNICEF/BDE, prior to starting the in-school training, are to organize meetings with school principals and pedagogues/psychologists, at which they would be informed about the Project, the course of activities and the expected outcomes. The openness for cooperation of the managing staff should be used in providing adequate support to teachers for taking part in training and in implementing the new-acquired knowledge.
- At school level, the project *Thinking Mathematics* is to be treated as a project for raising students' knowledge of mathematics at the whole school level, and in that sense to provide support to grade teachers in Grades 1, 2 and 3, by the mathematics teachers from upper grades, who might help them in raising the level of mathematics knowledge.
- To point out the importance of providing and using adequate teaching aids (related to the project concepts) and to give instructions how they could be provided (a large part of the manipulative teaching aids could be made of cheap materials and by the school itself).

With creators and administrators of training

- Care is to be taken that in-school training is to be carryed out through adequately balanced activities that would enable raising pedagogical as well mathematics knowledge of teachers.
- Training should discuss, promote and insist on active approach to teaching (using of multiple differentiated approach and various cooperative techniques, using manipulative teaching aids, tasks related to the environment, tasks that could be solved in many ways or have many solutions, activities through which students themselves should discover the concepts and the rules, etc.)[1].

1 Based on the Program for training in *Thinking* Mathematics developed by UNICEF, The Concept for Nine Year Primary Education and the Methods recommendations in mathematics' curricula for grades 1, 2 and 3.

- In the course of training, it is necessary to provide access to adequate manipulative teaching aids – so that teachers could sense the need and the usefulness of their application.
- Subject teachers together with grade teachers, (where there exists such opportunities), should be involved as trainers in the training.

INTRODUCTION

There is a common practice in well-planned action projects, and especially in those of wider extent, to make a baseline study prior to the project activities.

This report presents the findings of the current state prior to starting the project activities of *Thinking Mathematics,* that relate to the instruction of mathematics in the first cycle of grade teaching. The Project is carried out by the Bureau for Development of Education and UNICEF Office-Skopje, and on their behalf, the research study was conducted by the Macedonian Civic Education Center.

The conceptual framework of the study is set up on the basis of analysis of factors that could have impact on the effects of the project activities, and which could be found in studies of factors for effective teaching of mathematics, as well as of factors related to students' achievements in mathematics. Hence, the study is supposed to try to provide answers to the following questions:

- What are the attitudes of teachers to mathematics and to the teaching of mathematics?
- What are teachers' perceptions concerning the importance of certain contents and mathematical skills and about the expectations from their students?
- What is the level of pedagogical knowledge of teachers related to the teaching of mathematics prior to their in-school training?
- What is the level of mathematics knowledge of teachers, needed in carrying out the instruction of mathematics, on topics covered by project activities?
- What is the support to teachers, which is provided by the managing staff of the school in promoting the teaching of mathematics?
- What is the level of knowledge and understanding of students on issues and tasks in areas which are covered by the *Thinking Mathematics* program?

Together with it, comparisons were made of the state in the selected project schools and in the selected non-project schools.

The report is primarily aimed at managers and administrators of the project. Therefore, in it, most attention is devoted to the outcomes of the study, which are given in part three. The first and the second part of the Report contain baseline information about the Project and about the methodology of the study. Results are given in considerable details, in order to be used in planning project activities and in further evaluations. At the end, the most important statements and certain recommendations are given, as drawn by the authors of this study on the bases of the outcomes.

PART I – BACKGROUND

This part contains basic information about the project Thinking Mathematics and the project activities carried out so far. A Review of the research findings of factors in students' performance related to teachers, is also presented. It represented a starting-point for investigating the conditions before starting the implementation of the project activities.

1. BACKGROUND

1.1. Starting points

The former Yugoslav Republic of Macedonia, introduced in 2007/2008, a nine year primary education, based on the Concept for nine year primary education approved by the Minister of Education and Science.

The new curricula for the teaching subjects in primary education were developed according to the principles for primary education, set up in the Concept (above all, the principle of general education character of the primary school, the principle of quality of education and the international comparisons of knowledge, the principle of students active participation, and the principle of the best interest for the child).

The curriculum prescribes: the goals for a particular developmental period (grades 1-3, 4-6, 7-9), the objectives per grades, the particular objectives concerning selection of content, notions that are to be acquired and examples of activities and methods that teachers can use in achieving the set up objectives. The curriculum, also, offers didactic guidelines for teachers, as well as general instructions for monitoring student achievements.

In developing curriculum, consideration was also taken concerning the coverage of children in kindergartens at the age of five (in 2006 it was lower than 20%), so that, in setting up the goals and the expected outcomes, "lower expectations" were consciously supported in certain teaching subjects in the first developmental cycle (Grades 1, 2 and 3), compared to those in other countries of the European Union, with an idea that together with the implementation of the curriculum there it would immediately start with its ongoing evaluation, monitoring of students' achievements, as well as subsequent work with teachers concerning the methods of work with students, the way of reaching the goals and improving the understanding about the quality of instruction.

Parallel work will be run to achieve greater coverage of children in pre-school education and in introducing its compulosory status at the age of five.

1.2. Analysis of Curricula

The Bureau for Development of Education, having an interest in raising the quality of education, is striving continually to improve curricula, aiming to make students' achievements in the country be comparable to those of the students in other countries. Among other activities it supports carrying out projects focused to improving students' achievements in particular teaching subjects, including language literacy, numeracy and life skills.

Therefore, taking into consideration the results of Trends in International Mathematics and Science Study, which are among the lowest in Eastern Europe, there is an urgent need to make efforts to strengthen the country capacities to provide high quality instruction in mathematics in primary education. The Bureau for Development of Education in cooperation with UNICEF Office – Skopje, began, in the course of 2008, implementing activities related to the numeracy, aiming to develop curriculum which would enable students' higher achievement, and which would give teachers opportunities to assess students' development and to develop plans for achieving particular objectives and for the work in classes of mathematics in Grades 1, 2 and 3.

The activities carried out during 2008, were led by international experts (Judy Rohde, M.A. and Eric Wilmot, Ph.D.) and were focused on:

- making analysis of mathematics curricula for Grades 1, 2, and 3;
- making comparative analysis of curricula in other countries, in order to review the corresponding content, expectations and organization of instruction;
- establishing broader aims (standards) in mathematics;
- introducing the five topics (numbers and operations, algebra, geometry, measuring and analysis of data and probability) into grade 1-3 curricula;
- establishing indicators for the expected and supposed skills for each grade related to each of the topics.

The analysis, which confirmed, as it was envisaged by developing the new curricula, that our curricula (for Grades 1, 2, and 3), compared to those in other countries, have generally low expectations.[2] The analysis does not focus on the "weak" and the "good" points only, but, at the same time it gives recommendations to make more precise, to recompose and point out certain parts (aims, contents, didactical instructions, etc.).

The recommendations for all curricular topics refer, mainly, to bringing mathematics closer to students, in an acceptable way, by using their experience and the already acquired knowledge. So, for ex. in fulfilling the goals of ***getting sense of***

2 For example, when students get acquainted with numbers in Grade 1, they are expected to be able to count up to 10. Then, in Grade 2, they expand their knowledge up to 20. In Grade 3, they work with numbers up to 100.

numbers and operations it is recommended to enable students to use own strategies, to think about them and to share them with other students. It would help them not only to learn about their own thoughts and to learn from each other, but also, it would help the teachers to have insight into their process of thinking and to adapt their own instruction.

In the part related to ***algebra and functions***, in order to be competitive at international level, it is recommended that Macedonian students take part in activities which could improve their understanding about the models that are to be introduced and about the mathematical rules (for. ex. commutative, associative and distributive property) in solving problems.

The recommendation concerning studying ***geometry***, besides getting acquainted the students with plain geometrical forms and geometrical solids, refers to providing students with opportunities to describe the properties of forms they encounter. Also, students should be enabled to create symmetrical forms and be able to link notions of geometry with those of numbers and measurement.

In working with ***measuring*** it is recommended to establish relationship between content and other topics in mathematics, and in doing ***data analysis*** it is recommended to introduce data collecting and analysis in Grade 1, so that students, at the end of Grade 2 and in the course of Grade 3, would start to develop the skill to make assumptions and conclusions based on data, as well as the skill to use the basic notions of probability.

It is pointed out in the analysis, that one of the strong sides of curricula is that the stress is put on using commercial and self-made materials. For ex. in Grade 3, the play monopoly refers to using commercial materials, and the environment in teaching geometrical forms and the use of bundles of sticks, as an illustration of adding and subtracting tens, refers to using everuday self-made manipulatives.

According to the analysis, the part about assessing students' achievements represents a promising framework for collecting and analyzing performance and data about students' achievements, though it is necessary to state more precise standards and indicators for measuring achievement in order to make it possible to assess the achievement of each individual student.

1.3. Training of teachers

On the basis of the considerations from the analysis, an expert team developed a program to national trainers, aiming to train teachers to implement the given recommendations related to curricula in their instruction. In developing and implementing the entire training, the materials from "Thinking Mathematics for Grades 1, 2 and 3" were used – a program for training teachers (based on conducted international studies) prepared by the American Federation of Teachers, one of the

two larger federation in U.S.A. In addition, Dr. William Schmidt, the director of the Third International Mathematics and Science Study (TIMSS) in U.S.A, from Michigan State University, confirmed that "Thinking Mathematics" is a program that enables teachers to acquire profound knowledge which would change the instruction and enable their students to improve the performance. So, in the state of Minnesota, in U.S.A., the development of adequate standards in the mathematics curricula and the intensive training of teachers using the program "*Thinking Mathematics in Grades 1, 2, and 3"* and other similar materials, resulted in improving students' achievement in mathematics, in the period 1995-2007, within the framework of the TIMSS study.

The American Association of Teachers enabled the program "Thinking Mathematics in Grades 1, 2 and 3", to be used freely and without charge. The preparation of materials for mathematics was done by Ms. Judy Rohde, consultant, and Ms. Alice Gill, the author of "Thinking Mathematics".

The Bureau for Development of Education, in order to include better quality teachers, i.e. highly motivated teachers, to improve mathematics instruction, on the basis of open competition, made a selection of about fifty teachers (part of whose language of instruction is Macedonian, Albanian, Turkish or Serbian) and required from them:

- to attend the training structured in a total of 3 workshops;
- to implement the acquired knowledge in performing the instruction (with support of BDE advisers) and
- to train all teachers in the country teaching mathematics in Grades 1, 2, and 3 (first as trainers in their schools providing training to other teachers teaching in Grades 1, 2, and 3, as recommended by the advisers and the international team of trainers).

The main objectives of the training in *numeracy* are:

- to deepen the understanding about the quality of mathematics instruction by teachers;
- to implement the newly acquired knowledge in the instruction and so to become more efficient teachers;
- to improve students' achievements in the mathematics;
- to raise students' results in mathematics within the framework of the international comparative researches and studies.

Workshop Session One took place from 09.03 to 14.03. 2009, and it consisted of five-day training led by Ms. Judy Rohde, senior consultant, in cooperation with Ms. Mary Ellen Knappmiller and Ms. Marium Toure. The participants at the workshop were acquainted with the framework and the Ten Principles of Thinking Mathematics, and how they are used to facilitate the development path of students from counting to addition and subtraction.

Workshop Session Two took place from 15.05 to 20.05. 2009 and it consisted, also, of five day training led by the same consultants. The workshop dealt with the strategies that could be used to solve addition and subtraction textual problems.

Workshop Session Three took place from 22.06 to 27.06. 2009. The training was led by the same trainers. The topics of the workshop dealt with taking evidence (recording), questioning and assessing students, multiplication and division, data processing, solving problems that involve multiplication and division.

1.4. Dissemination of training and monitoring its results

After the training of the teachers and of the advisers from the Bureau for Development of Education, a phase of dissemination of training would follow, first at in-schools level from which the trained teachers come from. Teachers, under the mentorship of the advisers, are to implement the acquired knowledge from workshops at their mathematics classes, and then to carry out training with all teachers teaching in Grades 1, 2 and 3, in their own schools.

These teachers, with support of the Bureau for Development of Education, would administer training for all grade teachers in the country.

The implementation of the new approaches in raising numeracy would be monitored by the Bureau for Development of Education, from the aspect of quality of the performed instruction as well as from the aspect of the results achieved by students.

2. FINDINGS FROM PREVIOUS STUDIES

2.1. Studies of some relevant factors of students' achievements related to teachers

In undertaking educational reforms and in monitoring their effects, care is taken about factors having impact on the effects. Here, in addition to curricula and school facilities, the teacher has a central place. Hence, in monitoring the effects of the project *Thinking Mathematics*, in addition to the program for new approaches in learning mathematics, based on the Ten Principles, care should be taken about other factors related to the teacher which could have impact the effects of the program. A review of studies, related to certain characteristics of teachers that might be relevant to conducting and monitoring the activities of Thinking Mathematics, grouped mainly according to Koehler and Grouws' model (1992), and revised by Suriza van der Sandt (2007), is given below.

2.1.1. Teacher's mathematics knowledge

Researches related to teacher's knowledge refer, most often, to:

1. Content (mathematics) knowledge and to
2. Pedagogical knowledge (knowledge how to teach mathematics).

Findings from researchers show that:

- The way how the teacher performs the instruction., and its effects upon students' achievements, depends on how much teachers know the mathematics contents that they are teaching (Muijs & Reynolds, 2002; Ball & Bass, 2000, according to Van der Sandt S. (2007)). In 16 out of 18 analyzed studies, it is stated that there is a direct relationship between the content and the teaching practice (Horison Research (2008). In additions to the methodological limitations, in the majority of these studies, the findings give ground to make generalizations.
- The broadening of mathematics knowledge of teachers could result in change of the way of instruction and to deeper connection with pedagogical knowledge (Ormrod and Cole (1996) according to Van der Sandt S. (2007).

- Teacher's conduct in teaching depends on teacher's understanding of content that he/she is teaching, and of the understanding how students learn. (National Research Council (NRC, 2001), Van der Sandt S. (2007)).
- Knowledge and beliefs of teachers are related and have impact on students' achievement (Muijs and Reynolds (2002) according to Van der Sandt S. (2007)).
- One of the important factors in changing teaching practice, which should be taken in consideration, is the content knowledge of teachers (Clarke (1997) according to Bergeson T. (2000)).
- Pedagogical mathematics knowledge of teachers (measured directly, by a knowledge test) is directly related to students achievements in Grades 1, 2, and 3. (Hill, Rowan & Ball (2005).
- Mathematics knowledge is one the 12 factors related to the teacher which have impact on students' achievements (Schacter & Thum, (2004), според Goe L, L. Sticker (2008).

2.1.2. Attitudes and beliefs of teachers

Most often investigated attitudes of teachers, are those related to mathematics and the beliefs how mathematics should be taught. Research findings show that:

- Teachers beliefs related to the nature of mathematics and the general concepts how to teach mathematics, have strong impact on the teaching, due to the fact that on the basis of the beliefs teachers decide what they are going to teach, to what part they are going to give bigger importance, how to teach and how they would behave towards student's learning (NRC, (2001); Muijs & Reynolds, (2002); Schoenfeld, (2001) according to Van der Sandt S. (2007), Handal & Herrington, (2003); Kagan, (1992); Pajares, (1992) according to Yates S. (2006))
- Teachers beliefs are often a filter for the new knowledge and they could speed up or make slower the reform undertakins (Burkhardt, Fraser &Ridgway, (1990); Koehler & Grouws, (1992); Sosniak, Ethington & Varelas, (1991), according to Yates S. (2006))
- Each change in the teaching of mathematics should take into consideration teachers' beliefs and their changes (Swan (2006) according to Clarke J. (2008).
- Attitudes towards mathematics and to the teaching of mathematics have impact upon formation of students'attitudes towards mathematics, and through that to their achievements in mathematics (Ernest, (1989) according to Van der Sandt S. (2007).

There is a positive relationship between the degree of cognitiv-constructivistic orientation in pedagogical beliefs of teachers and students' achievements in solving problem tasks in Grade 1 (Peterson, Fennema, et al. (1989) and Staub & Stern, (2002).

PART II – METHODOLOGY

This part gives a brief description of the methodology used in the baseline study, i.e. information about the aims of the study, conceptual framework, indicators for the study, used instruments, sample, and collecting, processing and analysis of data.

Starting from the intention to monitor the quality of the implementation of approaches to numeracy in the program Thinking Mathematics in Grades 1, 2, and 3, as well as of the results achieved by students, it was necessary to provide relevant information about the baseline state before starting the project activities. In order to provide such information, we selected a methodological approach based on the following principles:

- Focus on the need for information related to evaluation of the project goals and the impact of the project activities;
- Providing basis for longitudinal monitoring and evaluation of the performance;
- Providing data which could be used in planning and managing subsequent project activities;
- Providing adequate base for making subsequent judgments and decisions for achieving better results, and especially at the level of outcomes and impact;
- Opportunities for replicable data in subsequent measuring, and
- Rationality from the aspect of timeline, human resources and budget.

We used the quantitative and the qualitative approach in the study.

1. OBJECTIVES OF THE STUDY

We conducted the study prior to starting the training of teachers from the project schools. On the basis of the goal of the Project: *to raise the level of knowledge and skills of teachers, to improve students' achievements in mathematics,* the intention of this study is to provide relevant information about the baseline state which together with the subsequent monitoring and the evaluation of the project activities, would serve as a basis in measuring the impact of the project – better quality instruction and higher achievements of students.

The objectives of this study are:

1. To provide information about the baseline state of mathematics' teachers concerning:
   - their attitudes to mathematics, teaching and learning mathematics (pedagogy in the instruction of mathematics) and
   - their mathematics content knowledge and pedagogical knowledge relevant for teaching mathematics in *Thinking Mathematics.*
2. To provide information about students' achievements at the end of Grade 3 on on test's items in content domains covered by the program Thinking Mathematics.
3. To survey the activities for improving the teaching of mathematics at school level.
4. To assess the specific needs for training.

2. CONCEPTUAL FRAMEWORK

In the investigations of the baseline state, on the bases of the surveys of factors that have considerable impact on students, we decided to examine the ongoing state of the projects schools and the difference between the project and the non-projects schools in relation to:

Teacher related factors

- Mathematics knowledge
- Pedagogical knowledge in teaching mathematics
- Attitudes towards mathematics and the teaching of mathematics
- Attitudes towards learning mathematics and the pedagogical approaches in teaching mathematics
- Expectations from students
- Familiarity with curricula
- Training of teachers to use interactive methods of instruction (instruction directed to student)

Factors related to socio-economic environment

- Parental education

Factors related to school environment

- Peer support of the school staff
- School equipment for the teaching of mathematics

In addition to that, the familiarity and the comitment of the managing staff towards the project *Thinking Matheatics* was examined also. More concrete:

- The satisfaction with the ongoing state in the achievements of mathematics
- The activities to improve the teaching of mathematics
- The support to teachers in teaching mathematics
- The opinions to innovative projects
- Familiarity and preparedness to support the project activities

3. INDICATORS

I.1 – Teachers' current understanding of learning and teaching and its the compatibility with the way of teaching based on the Ten Principles in *Thinking Mathematics* (Understanding of teaching and learning mathematics)[3]

- Attitudes towards learning mathematics and towards teaching mathematics based on the Ten Principles of *Thinking Mathematics.*
- Attitudes towards mathematics and the teaching mathematics.
- Pedagogical knowledge of teachers in accordance with the approach in *Thinking Mathematics.*
- Teachers' expectations concerning the mathematics' achievements of their students.
- Familiarity with mathematics curricula for the next cycles in primary education.

I.2 – Teachers on-going knowledge and understanding, related to the key mathematics concepts of the program *Thinking Mathematics* (Teachers' knowledge of mathematics)

Mathematics knowledge of teachers and their understanding of:

- the number concept
- the four basic arithmetical operations and their characteristics
- the textual tasks in solving problems.

I.3 – Support to the teaching of mathematics by the school management staff (Support to the changes in the teaching of mathematics)

1. To what extent the school principals and the pedagogues/psychologists
   - are satisfied with the achievements in mathematics in the grade teaching cycle
   - believe that the students in the grade teaching cycle could have higher achievements in mathematics.
2. Support given to the teachers in the instruction of mathematics.

3 The indicator is given as formulated in the Logical framework of the investigation (Supplement 1), and the shortened names that are later used are given in brackets.

3. Equipment with manipulative/teaching aids for the instruction of mathematics.
4. Preparedness of the school principals and pedagogues/psychologists to give support to the implementation of new approaches in the teaching of mathematics.

I.4 - Achievements of students

Students' achievements on the mathematics test, which contained items that measure the conceptual and procedural knowledge, understanding and application of natural numbers, the four basic operations, as well as in solving textual tasks and problems.

4. METHODS AND INSTRUMENTS FOR DATA COLLECTION

In accordance with the defined indicators, the following sources and methods for collecting data were used in this investigation:

1. **Teachers' survey**

Using tests, questionnaires, attitudes scales and vignettes for mathematics teaching situations specially prepared for this study, we collected data about the ongoing math and pedagogical knowledge of teachers for the instruction of mathematics, as well as data about their views on learning and teaching mathematics, the expectations from their students, familiarity with curricula and the preparedness of teachers to use interactive methods in the teaching (teaching oriented to students).

2. **Assessment of students' math knowledge and understanding**

Using test developed for this study, administered at the beginning of Grade 4, we collected data about students' knowledge acquired at the end of Grade 3 in the domains od numbers and four basic mathematics operations, and their skills in solving textual tasks and simple problems.

3. **Interview with school principals and pedagogues/psychologists**

Using a protocol for semi-structured interview, we collected information about the support given to teachers in carrying out the instruction, the equipment and the professional development of teachers related to the teaching of mathematics, as well as about the familiarity and the conduct of the managing staff to the Project *Thinking Mathematics*.

Using a specially developed protocol, we collected *data about the schools*: about the size of the schools and about parental education.

The majority of the collected data are quantitative, in order to enable objective comparisons between the project schools and the non-project ones, prior to starting with project activities, as well as for subsequent comparisons during the course of the Project.

A brief description of the content of each of the instruments is given below. More detailed information about the instruments are given in Appendix 5.

Instrument	Brief description
Questionnaire for teachers	The questionnaire consists of three parts: **The first part contains:** - 5-level Likert's *Scale of attitudes to mathematics and to the teaching of mathematics* containing 23 items - 5-level Likert's *Scale of attitudes related to learning of mathematics and to pedagogical approaches in the instruction of* mathematics based on the Ten Principles in the approach to *Thinking Mathematics* containing 30 items - 3 questions related to: teaching of mathematics, the training of teachers in various projects that promote new approaches in the grade teaching and the classroom equipment for the instruction of mathematics; **The second part contains:** - *Test of teachers' mathematics knowledge* containing 15 items, which In order to give response to, teachers should make use of their general mathematics' knowledge and the mathematics' knowledge that is important for the teaching of mathematics in Grades 1, 2, and 3. - *Test of pedagogical knowledge of teachers* which contains 11 teaching situations (vignets) to which teachers should give response to, using their own pedagogical knowledge, and - two questions about teachers' expectations from students at the end of Grade 3. **The third part contains** - Three questions about teachers, background data.
Test for students	*Test for students* has 19 items which measure knowledge and skills in the domains: - Number concept – 5 items; - Operations (addition, subtraction, multiplication and division) and the characteristics of operations – 10 items; and - Problem situations involving operations, models and work with data) – 4 items. 9 multiple-choice items, 6 items with brief short answer, and 4 open ended items, which required complete procedure in giving responses were used, .
Reminder for the interview	Two groups of questions were given in the reminder for the interviews with the school management staff: - A group of 6 questions related to the teaching of mathematics in the school, and - A group of 7 questions related to the familiarity with the Project and the preparedness to give support to teachers of the project scools.
Protocol	Using a protocol, we collected data from each school about the number of classes and sudents in Grades 1, 2 and 3, and data about student's parental education in Grade 4 (the new one).

Prior to developing the final version of the questionnaire for the teachers and the test for the students, they were piloted in one project school, which was not included in the sample.

5. SAMPLE

5.1. Selection of sample

One of the research challenges was to identify two samples – one for the project schools and one for the non-project ones. The population of the project schools consisted of 35 schools, which took part in the Project upon applying to an open competition for teachers – as subsequent trainers in the Project. It shows that the project schools are not the representative of the entire population of schools in the country, nevertheless, the project, at the end, has an aim to cover all the schools in country. We had a dilemma, to measure the starting state with a representative sample of schools, teachers and students at the national level and in that way to provide highly reliable basis for comparing the effects where all the schools would be included, or to develop a sample comparative (parallel) to the starting project sample, which would provide more reliable comparing of effects from the project activities in the course of the Project and indicators for intervention during its implementation, but finally we decided to choose the second approach. So, two comparable cluster samples were selected:

1. Sample of project schools and
2. Sample of non-project schools

In developing the sample of project schools, care was taken that it should represent the population of primary schools in country in relation to:

- geographical coverage;
- location of schools (urban – rural) and
- language of instruction (Macedonian - Albanian).

15 project schools were selected. Due to the limited number of project schools and their characteristics concerning the mentioned factors (the larger part are urban and with Macedonian language of instruction), they could provisionally be considered as a representative sample related to the entire populations.

Then, we deliberately made a selection of parallel schools, taking into consideration that they be identical concerning the geographical coverage (from same urban and rural places) and the language of instruction, and similar concerning the social background of students (which was later checked by their parental education) with that of the project schools.

The sample of schools is given in the following table:

Table 1. *Sample of schools according to language of instruction*

Geographical coverage	project schools			non-project schools			Total
	Macedonian	Albanian	mixed	Macedonian	Albanian	mixed	
Urban	9	1	2	9	1	2	24
Rural		2	1		2	1	6
Total	9	3	3	9	3	3	30

In order to provide comparison of the changes in the course of the Project, samples were compared in relation to parental education of children in the grade teaching cycle.

Table 2. *Education of mothers of students in project schools and in non-project schools*

Education of mother	Project				Non-project			
	Macedonian		Albanian		Macedonian		mixed	
	No	%	No	%	No	%	No	%
not completed primary	49	4,2	1	0,1	31	4,0	16	1,7
Primary education	123	10,5	783	93,5	74	9,1	588	63,2
Secondary education	743	63,2	30	3,6	470	57,4	293	31,5
Higher and university	260	22,1	23	2,8	242	29,5	33	3,6
Total	1175	100,0	837	100,0	819	100,0	930	100,0

Table 3. *Education of fathers of students in the project schools and in the non-project schoo ls*

Education of father	project schools				non-project schools			
	Macedonian		Albanian		Macedonian		Albanian	
	No.	%	No.	%	No.	%	No.	%
not completed primary	23	2.0	0	0,0	21	2,6	5	0,5
Primary education	141	12,0	671	80,4	91	11,1,	365	39,0
Secondary education	468	65,5	125	15,0	482	59,0	499	53,3
Higher and university	241	20,5	39	4,6	233	27,3	67	7,2
Total	1173	100,0	836	100,0	817	100,0	936	100,0

Parental education in the project schools with Albanian language of instruction is considerably higher than that in the non-project schools. There are no considerable differences in parental education in the project and the non-project schools with Macedonian language of instruction.

5.2. Selection of students

In each of the selected schools, 20 students from Grade 4 (the new one4) were selected by random sample. The students, at the beginning of Grade 4, were the population closest to the target group, and which is expected to demonstrate most evidently the impact of the program (at the end of Grade 3). The investigation was conducted on 598 students, out of 600 selected students.

5.3. Selection of teachers

In each of the schools, 10 grade teacher were selected by random sample. The population of grade teachers is the one that would undergo the training and would teach in the first cycle of primary education (Grades 1, 2, and 3). The survey was conducted on a total of 299 teachers. The following table presents their sex, education and working experience structure.

4 These students are at the age of nine.

Table 4: *Characteristics of the sample of teachers*

Characteristic		project-schools		non-project schools	
		BO 2009		BO 2009	
		number	%[5]	number	%
Gender	female	113	75,3	116	77,9
	male	22	14,7	20	13,4
Education	higher - grade teaching	40	26,7	37	24,8
	university - grade teaching	78	52,0	71	47,7
	university – pedagogy	16	10,7	27	18,1
Working experience	up to 5 years	9	13,3	8	6,0
	5 – 10 years	20	14,8	10	7,4
	11 – 20 years	46	34,1	66	49,3
	over 20 years	60	27,8	50	27,2
Training in projects2	Active teaching	113	75,3	106	71,1
	Step by Step	41	27,3	76	50,3
	PEP mathematics	23	15,3	29	19,5

According to the controlled characteristics, there are no significant differences between the project and the non-project schools concerning the gender and the education structure, as well as the working experience[5] of teachers. Considerably higher percent of teachers from non-project schools were trained for the project *Step by step. There is no differences in training for other projects.*

5.4. Selection of school principals and pedagogues/psychologists

The sample of the school management staff consisted of school principals and one pedagogue or psychologist from the project schools. However, due to the great interes and the readiness to take part in the interview with the members of the school management team, the sample is a bit larger. The data were already collected and so we decided to include them in the processing. We processed the data from the interviews with:

- 15 school principals and 2 deputy principals
- 8 psychologists
- 11 pedagogues

5 The difference in the category of working experience 11-20 years, in fact is due to the large concentration 15% of teachers with working experience of 11 and 12 years in the non-project schools.

6. DATA COLLECTION, PROCESSING AND ANALYSIS

Collection of data was conducted by selected advisers from the Bureau for Development of Education, who were involved in the project *Thinking Mathematics in Grades 1, 2 and 3.* At first, they went through a one-day training[6] organized for them, at which were agreed the procedures and deadlines upon about the administration of tests for students and the questionnaires for teachers, as well as for the interview with school principals and pedagogues/psychologists.

The collection of data began on November 4th, and ended on November 17th, 2009.

After the collection of data, we did the coding of the filled-in instruments, the marking the tests for teachers and for students, the input of data and the qualitative analysis of the recorded responses from the interviews.

Data were entered into Excel program, and then were processed by using:

- TIA plus program, for the tests with the teachers and the students. It checks the psychometric characteristics of the tests and the attitude scales, shows achievements on the tests, and enables making comparisons between the project and the non-project schools in relation to the outcomes from the tests and the attitude scales, and
- SPSS program in processing the responses to the questions and making comparisons of the responses to the questions between the project and the non-project schools.

Quantitative and qualitative data are analysed in relation to the defined indicators, and in some categories by using topic analysis.

6 Appendix 4 gives the guidelines for the researcher, that were developed for this training.

PART III - OUTCOMES

Part three presents the data obtained from the survey. They are presented according to the defined indicators. The data represent a cross-cut of the state of the relevant factors in monitoring the effects of the program in the project and the non-project schools, and they could serve well for subsequent longitudinal monitoring. They are an indicator of the starting state, but due to the limiatations of the sample, they cannot be generalized for the entire population.

1. UNDERSTANDING OF LEARNING AND OF TEACHING MATHEMATICS

The currently understanding of how pupils learn mathematics and how teachers teach mathematics was examined using three instruments. A brief description of each of them, and the results obtained from the survey, are given below.

1. 1. Attitudes of teachers to learning mathematics and to teaching based on the Ten Principles

METHOD OF MEASURING

Attitudes of teachers were measured by a Likert's 5-point scale containing a total of 30 statements:

- 15 related to learning mathematics, and
- 15 related to teaching mathematics

Items were formulated in such a way as to express attitudes in accordance with the Ten Principles of *Thinking Mathematics* or attitudes opposite to the Ten Principles.

Examinees were asked to denote the level of agreement with each statement at the 5 point scale (from 1 – I don't agree at all, up to 5 – I fully agree).

1.1.1. Attitudes of teachers to learning and teaching mathematics

Results are presented by mean of the degree of acceptance by all the teachers, and separately by the teachers in the project and in non-project schools, of the scale as a whole, and of the subscales. Special comments are given to statements which to a higher and lower degree are accepted in relation to the other statements.

- Generally speaking, the examined teachers were inclined to accept statements that describe the learning of mathematics in accordance with the understanding of learning in *Thinking Mathematics* and with the practice adjusted to the Ten Principles in learning mathematics. (The Mean (M) for the entire sample is 86,84 which is significantly higher above the theoretic M = 75, and which could be interpreted as indefinite attitude. Certain inclination to the positive part of the scale is expected due to the fact that the pedagogical knowledge of teachers, and particularly that acquired during the

complementary training[7] enables them to recognize which approaches are professionally more acceptable the teachers to be shown in a positive light. [8]

- Teachers in project and in non-project schools do not differ in the degree of accepting the statements in the scale of attitudes (M for project schools = 87,32; M for non-project schools = 86,36 – the difference is not statistically significant). The degree of acceptance of approaches adjusted to the Ten Principles in both groups is above the theoretical average value of the scale (75)
- Teachers in project and in non-project schools do not differ in the degree of accepting the statements in the subscales that relate to the beliefs concerning how children learn mathematics (M for project schools =39,88; M for non-project schools =38,67 - the difference is not statistically significant). The degree of accepting the statements based on the Ten Principles about how children learn mathematics is insignificantly below the theoretical mean (37,5) in both groups.
- Teachers in the project and non-project schools do not differ in the degree of accepting the statements in the subscale that relate to the description of their own practice in teaching mathematics (M for project schools = 40,47; M for non project schools = 40,42 – the difference is not statistically significant). The degree of accepting the statements related to their practice of teaching mathematics based on the Ten Principles in both groups is above the theoretical mean (37,5). The difference related to the theoretical mean in this subscale is a bit larger, than that in the subscale related to the understanding of learning mathematics, probably because it was easier for them to recognize the descriptions of preferable teaching practice.

The average degree of accepting for half of the statements (15) is about 3 (3 ± 0,5) on a scale from 1 to 5. The descriptions of the statements which are at an average less or more accepted, are given below.

The most characteristic statements about the teaching practice that are not in accordance with the approaches in *Thinking Mathematics*, with which the teachers in the project and non-project schools agree to a very large degree, refer to:

- the feeling of obligation concerning the consistent implementation of curriculum[9];
- the role of teacher as an instructor/conveyor of knowledge;
- the time frame for giving negative feedback.

The most characteristic statements concerning the understanding how children learn mathematics, and which are not harmonized with the approaches on

7 More than 3/4 of the teachers responded that they had been trained in some of the projects promoting similar approaches as those in Thinking Mathematic,(see page 23 in this Report)

8 Almost all the surveys (for. ex. TIMSS, PIRLS, The National Assessment) confirm the expressed inclination of teachers in country to give socially preferable answers.

9 - A brief description of the content of the statement is given here. The exact formulations are not given for the reason that they might be used later in the scale of the Project.

Teachers in project and in non-project schools, agree to a very large degree on issues that describe their teaching as an instruction which:

- makes use of life experiences of children and of simulations of life situations;
- provides to them, very often, feed back, and
- practices self-assessment and peer assessment.

which is in accordance with the approaches promoted by *Thinking Mathematics.*

In relation to the understanding how children learn mathematics, teachers from both groups agree, above the average, only on the issue that the way of representing numbers in various ways helps them, which is, also, in accordance with the approaches promoted by *Thinking Mathematics.*

CONCLUSION

- The fact that there are no statistically significant differences found between the teachers in the project schools and those in the non-project schools, concerning the understanding of learning mathematics with children and in the accordance concerning the descriptions of the teaching practice related to the Ten Principles, will enable easier monitoring of eventual changes during the Project.
- The results point out to the ongoing understanding of the importance of using various ways in presenting numbers, using of life experiences and of formative assessment in to teaching, but, also to the known weaknesses in the teaching practice such as: teaching that is too much dictated by curricula (content) instead of by aims, it is insufficiently differentiated in the approach, there is too much content centered and traditional teaching style in the instruction, promoting of mathematigcs as a discipline which is predominantly a sum of notions, rules, procedures, definitions, etc. These findings could serve as an indicator about the areas to which more attention is to be devoted during the Project.

1.2. Attitudes of teachers to mahematics and to the teaching mathematics

METHOD OF MEASURING

In general, teachers' attitudes to mathematics and to the teaching of mathematics are measured on a Likert's scale , which consisted of a total of 23 items related to mathematics and to the teaching of mathematics.

Statements are formulated in a way as to express satisfaction and dissatisfaction from learning and being engaged in mathematics, and satisfaction or dissatisfaction from teaching mathematics.

Examinees were asked to denote the level of agreement with each statement at the 5 degree scale (from 1 – I don't agree at all, up to 5 – I fully agree).

1.2.1. Attitudes of teachers to mathematics

Results are presented by mean of the degree of accepting the statements by all of the teachers, and separately by teachers from project and non-project schools.

Separate comments are given to statements which are accepted, to a higher or lower degree, in relation to the other statements.

- The examined teachers were inclined to a high degree to accept the statements which describe them in a positive light in relation to their interest in mathematics, performance in mathematics, when they had been students, as well as about their abilities and interest in the teaching of mathematics. The mean (M) for the entire sample of teachers is 86,91, and the theoretical mean (TM), which could be interpreted as a non-defined attitude (neither accepting, nor not accepting) is 55,75. Certain inclination to the positive part of the scale was expected, but not to such a high degree, taking into consideration the impression that many people do not have positive experiences with learning mathematics and do not like mathematics[10].
- Teachers in the project and in the non-project schools do not differ in the degree of accepting the statements at the scale. (M for the project schools = 86,85; M for the non-project schools = 86,96 – the difference is not statistically significant).

To a somewhat lower degree (average accepting is around 3 – neither accepting nor no accepting) are accepted the statements of the type: I am often asking myself ... , and then they show dilemmas about the way and the successfulness in explaining mathematics content in the instruction.

10 In our country, there are no surveys about neither part of the population concerning the distribution of responses on such a scale.

CONCLUSION

- Positive attitudes to mathematics and the teaching of mathematics should represent a good ground in improving the teaching practice in mathematics.
- The fact that there are no statistically significant differences in attitudes, enables to keep this factor under control and not to represent a factor of proficiency of the Project. Namely, according to theory and investigations, it is considered that attitudes to mathematics and to the teaching of mathematics do not change easily with adults that have already had experience in learning and teaching mathematics (for ex. Block & Hazelip, 1995; Kagan, 1992, according to Novotna and all (2006).

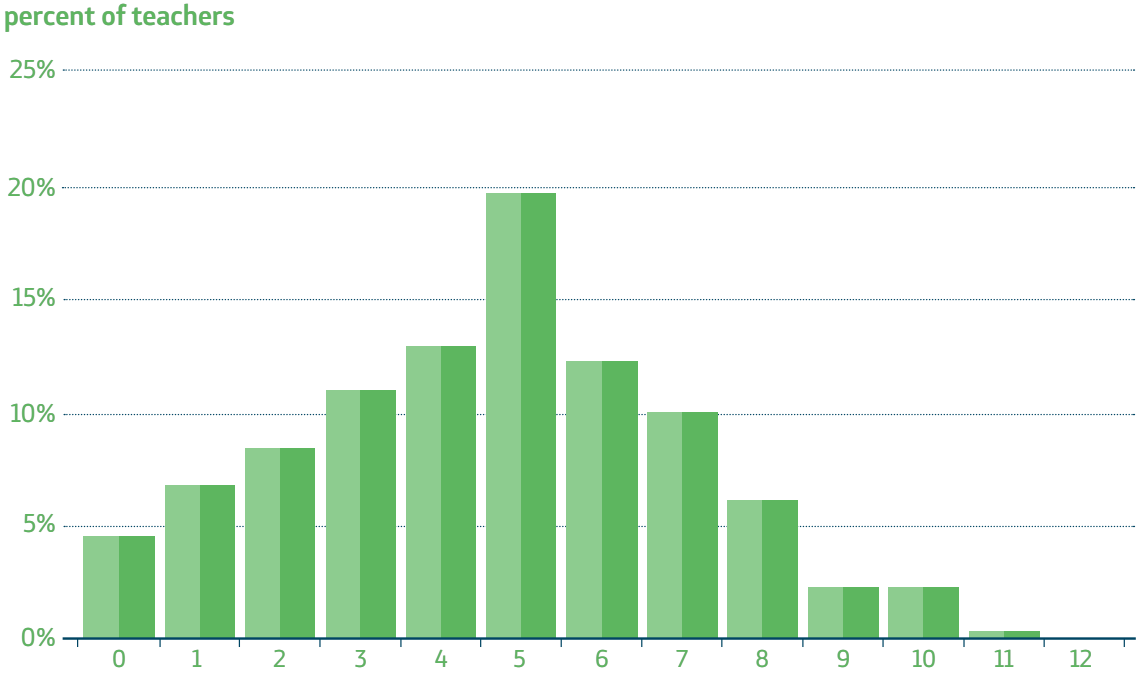
1.3. Pedagogical knowledge of teachers

METHOD OF MEASURING

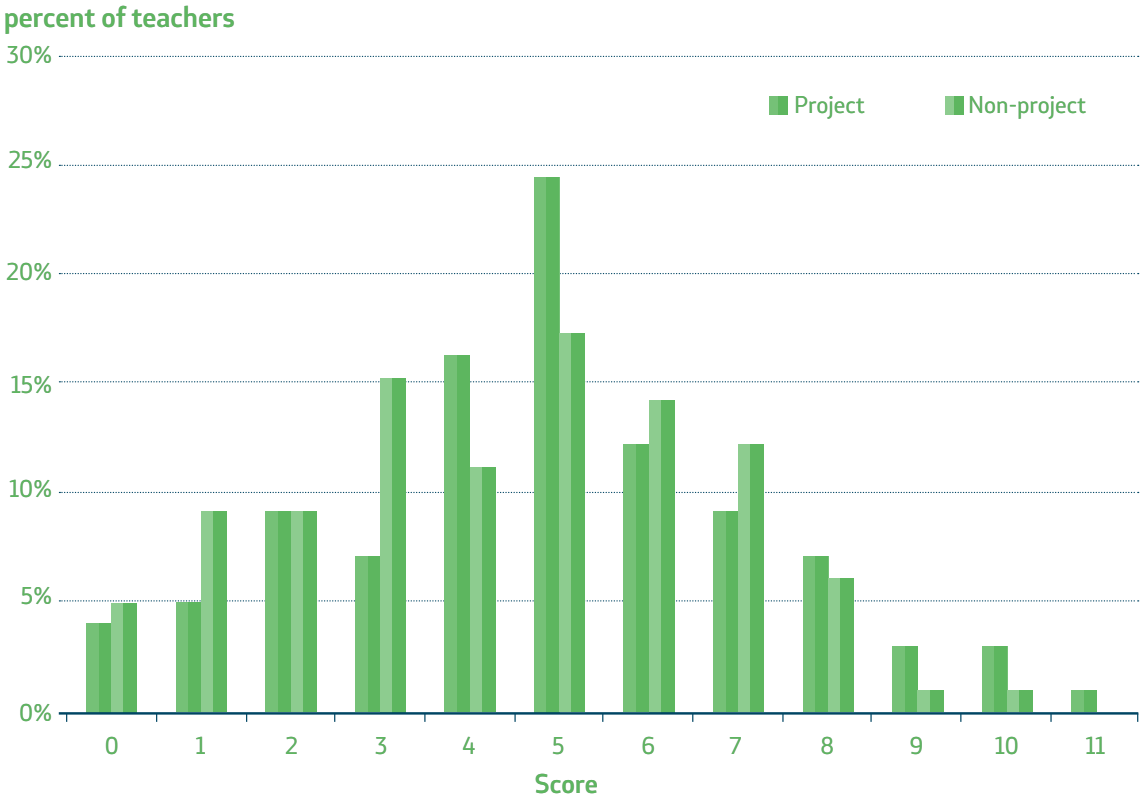
Pedagogical knowledge of teachers was measured by a *Test for the pedagogical knowledge of teachers,* which consisted of 11 teaching situations (one of which was a cluster one) to which they were asked to give responses, using their pedagogical knowledge.

1.3.1. Achievements of teachers at the test in pedagogical knowledge

The average result at the test in pedagogical knowledge of **all the teachers** is 4,62 (maximum possible is 14), i. e. the average percent of solving is 32, 99. The highest achieved score is 11 and that is for only one teacher, and the highest is the percent of teachers (20.40%) having a score of 5. The achievements at the test have a distribution inclined to the left – which seems to look like a normal one, but the highest scores of 12-14 have not been achieved, at all.

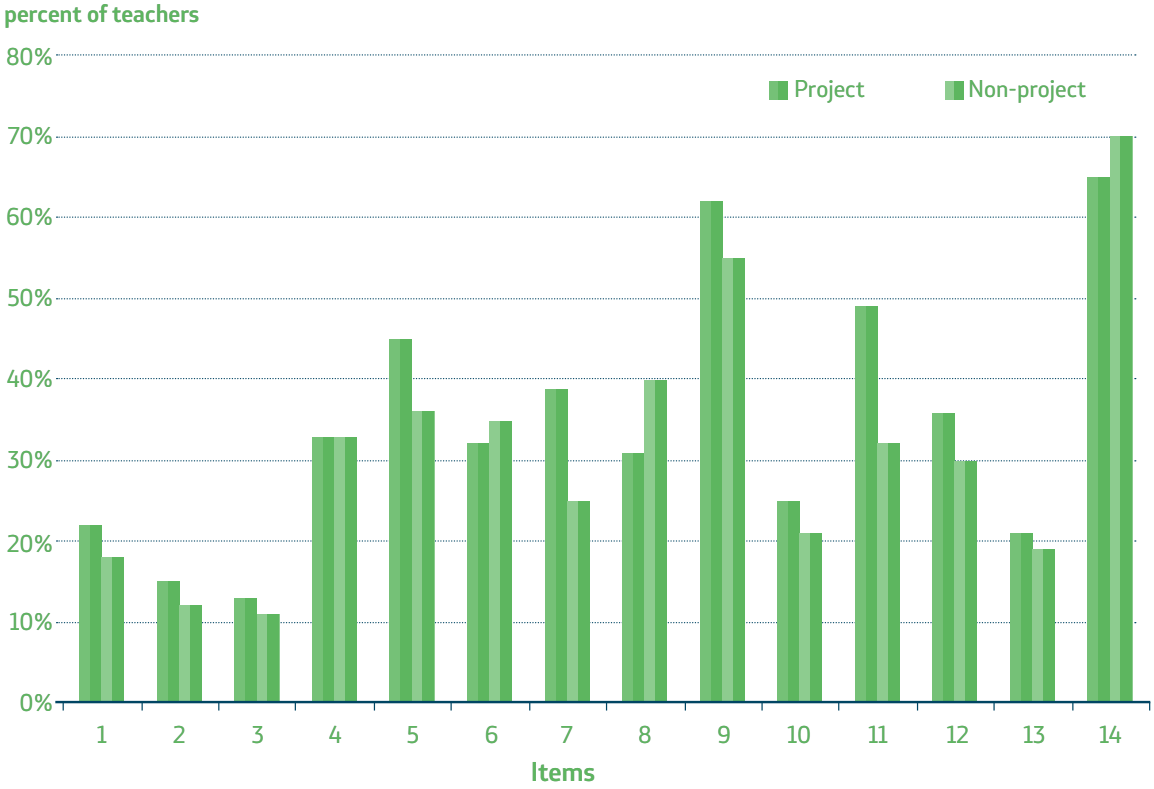


Graph 1. *Result of all teachers on the test of pedagogical knowledge*



Graph 2. *Result of teachers from project and non-projects schools on the test of pedagogical knowledge*

The graph below presents the results of the pedagogical knowledge of teachers from project and non-project schools on each item of the test.



Graph 3. *Teachers' results from project and non-project schools related to the items of the test in pedagogical knowledge*

The table gives descriptions of the requirements of the items and the relationship with the Ten Principles promoted by the project *Thinking Mathematics*. The number of the item in the table corresponds to the number of the item in the graph given above..

Table 5. *Relationship of the test in pedagogical knowledge with the Ten Principles*

Item	Description	Relationship with teh Ten Principles
1	Reaction to a response by puzzled student/ again similar sub-question	Formative assessment – feed back
2	Reaction to a response by puzzled student/leading question	Formative assessment – feed back
3	Reaction to a response by puzzled student/permission for repeated thinking	Formative assessment – feed back
4	Reaction to a response by puzzled student/paraphrasing the reply	Formative assessment – feed back
5	Reaction to good response given by weaker student	Assessment for learning and requiring explanation for mathematical thinking

6	Introducing multiplication of a two-digit number	Using various strategies
7	Strategies in teaching multiplication	Using manipulative aids
8	Unusual way of adding 2 three-digit numbers with "signing"	Accepting and motivating the use of different procedures in solving
9	Support to a student having problems in adding with crossing	Using intuitive knowledge and use of manipulative aids
10	Reaction to a mistake in one of the steps in solving a task in two steps	Balancing conceptual and procedural knowledge, solving textual tasks
11	Different way of determining a sum of 3 two-digit numbers	Accepting different strategies in doing the tasks, requirement of explanation for the solution
12	Change of topic planned for a class hour	Adjusting the time-line in introducing the content adequately to the interest of students
13	Explaining grouping of tenths and units	Selection and use of adequate manipulative aids
14	Presenting multiplication	Using manipulative aids and graphic presentation, linking of concrete and symbolic presentation

On this test, characteristic are the answers to the item number 10.

Task

Marko, a student from Grade 3, did this task: Ivan wants to share the chocolate bar with Ace and Ana. The chocolate bar has 6 rows of 4 cubes. How many cubes would each one of them get?
He did the task in the following way:

$6 \cdot 4 = 24$ $24 : 3 = 7$

What should be best for the teacher to do? Choose one answer.

A) To check if Marko knows that division is an opposite operation to multiplication **1**

B) To ask Marko to draw it on a piece of square paper **2**

C) To tell him to check the response **3**

D) Something else..... **4**

(write what)

The largest percent of teachers - 41%, sellect A as the best response. The response which we consider to be the best one – B, was chosen by 23%. From these responses, it can be stated that:

- Less than a quarter of the teachers (those 23%) noticed, that the student in his/her response showed that he/she has understood the problem situation, that he/she presented it successfully with a number sentence, and that he/she possesses both conceptual and procedural understanding of multiplication/division.
- Teachers did not notice that in the given situation the rule that division is an opposite operation to multiplication is not important, but they noticed only the number sentence and the results, and as an acceptable answer they chose A.
- The final Marko's result is not correct, but it would be better for Marko to see his mistake by himself – i.e. by drawing it in order to check his solution.

CONCLUSION

- Pedagogical knowledge of teachers is bellow the required level in teaching according to the Ten Principles of *Thinking Mathematics*.
- Teachers from project and non-project schools do not differ in their achivements on the test for pedagogical knowledge.

1.4. Teachers' expectations from students related to the achievements in mathematics

METHOD OF MEASURING

Teachers' expectations related to mathematics' knowledge of students were measured with 3 questions:

At the end what grade, they consider that the students could determine each of the given 5 mathematical tasks (1. Task with adding up to 10 with counting in; 2. Adding within the second tenth; 3. Adding two two-digit numbers with crossing the tenth; 4. Problem task in 2 steps involving adding up to 20; 5. Noticing the simple rule and finding the numbers that are missing in a given subtraction and addition);

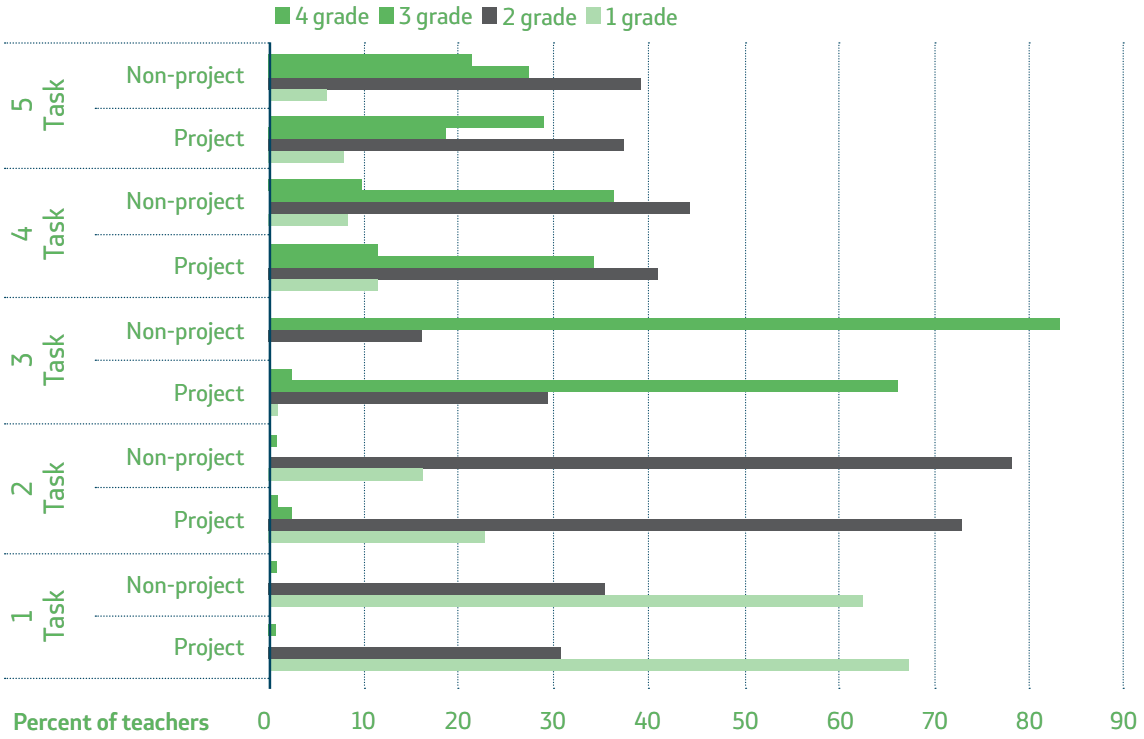
The importance they give to meeting the 13 objectives in mathematics until the end of Grade 3;

To what extent they agree with the statement that students in their classes could achieve more than requirements prescribed by curiculum.

Results are presented in arithmetical means or percents and comparisons are made between the answers of the examinees from the project and the non-project schools.

1.4.1.Expectations from students in doing tasks with different levels

The opinions of teachers about when the students would be able to do a particular type of task are an indicator about their own teaching practice, their experience, and their expectations. Their replies to five typical tasks are given below..



Graph 4. *Expectations about the ability in doing different tasks in particular grades*

Description of tasks:

1. Task - Adding to 10 with counting in

2. Task - Adding within the second tenth

3. Task - Adding two two-digit numbers with crossing the tenth

4. Task - Problem task in 2 steps which includes adding to 20

5. Task - Discovering the simple rule and finding the missing numbers in a given subtraction and addition

The first task: The majority of teachers (about 65%) both from project and non-project school consider that the task could be done by the majority of Grade 1 students, while about one third – of Grade 2 students.

Due to the fact that the percent of teachers consider that by the end of Grade 2 the majority of students would be able to do a task in which by counting students should determine the sum of two numbers less than 10, it could be stated that teachers expectations are low. Curriculum for Grade 1 prescribes activities such as

counting objects, adding and subtracting 1, as well as solving problem situation by adding and subtracting.

The second task: The majority of teachers chose Grade 2 (about 75%), as a grade in which the majority of students would do this task, and about 20% consider that students would do it in Grade 1.

Though, the majority of teachers consider that this task would be done by students at the end of Grade 2, this is a task that could be done successfully by Grade 1 students (by counting in, for example, or by using manipulatives).The choice at the end of Grade 2, is probably due to the fact that the curriculum for Grade 2 explicitly contains objectives related to operations of adding and subtracting up to 20.

The third task: Corresponding to the objectives of the curriculum and the requirements of the tasks, 66% of the teachers in project schools and 83% of the teachers in non-project schools consider that the end of Grade 3 is the period when this task could be done by the majority of students. This is the only task in which the difference in expectations of teachers from project and non-project schools differ considerably. Teachers from non-project schools have lower expectations, and the majority of them consider that this task could be done by the Grade 3 students.

Putting the requirement of this task in adequate context (measuring length, playing a shop, or simiar) or using manipulative aids or drawings, would enable Grade 2 students to do such tasks, who understand the concept of number.

The fourth task: There is great dispersion of the responses to this task both by teachers from project and from non-project school. Most often, it is expected to be done in Grade 2 or in Grade 1.

The great dispersion of answers, probably comes from the way in which teachers work with students. According to its content, the needed "mathematics" to do the task, by a choice of teaching aids and their use, the task could be done by Grade 1 students, because here the point is on addition where the sum is greater than 10, so teachers probably chose Grade 2 (and similar examples are also mentioned in the curriculum for Grade 2, in the part of activities and methods). Teachers who consider this task as a problem situation which could be done by an in-between question or by setting and doing a numerical statement, chose Grade 3 (prescribed by the curriculum for Grade 3) or Grade 4.

The fifth task: With this task, there is least accord about when students would be able to do it, though it dominates in Grade 2, but frequent are also the responses for Grade 3 and 4.

Similar is the situation as with task four, and the teachers probably decided about it in correspondence with their way of work and the experience that they have with students. Corresponding to the requirements of the curriculum (from the aspect of content: adding and subtracting to 10, i.e. 20) it refers to a task that could be done successfully by students from Grade 1 and Grade 2. The task requires to notice the rule, which is very simple and evident, but one could suppose that such tasks are rarely offered to students, and when they are used, the teacher generally reduces

the cognitive level of the task by giving "small help", by directions or sub-questions, instead of using manipulative aids.

CONCLUSION

- Expectations related to the period when the majority of pupils would be able to do particular tasks are in accord with the curriculum requirements and are lower than the abilities of students at particular age, provided that more adequate approaches in instruction are used (according to the findings in reference literature).
- Generally, there is no difference in the expectations of teachers from both project and non-project schools, which is a sound base for monitoring the changes as a result of the project activities.

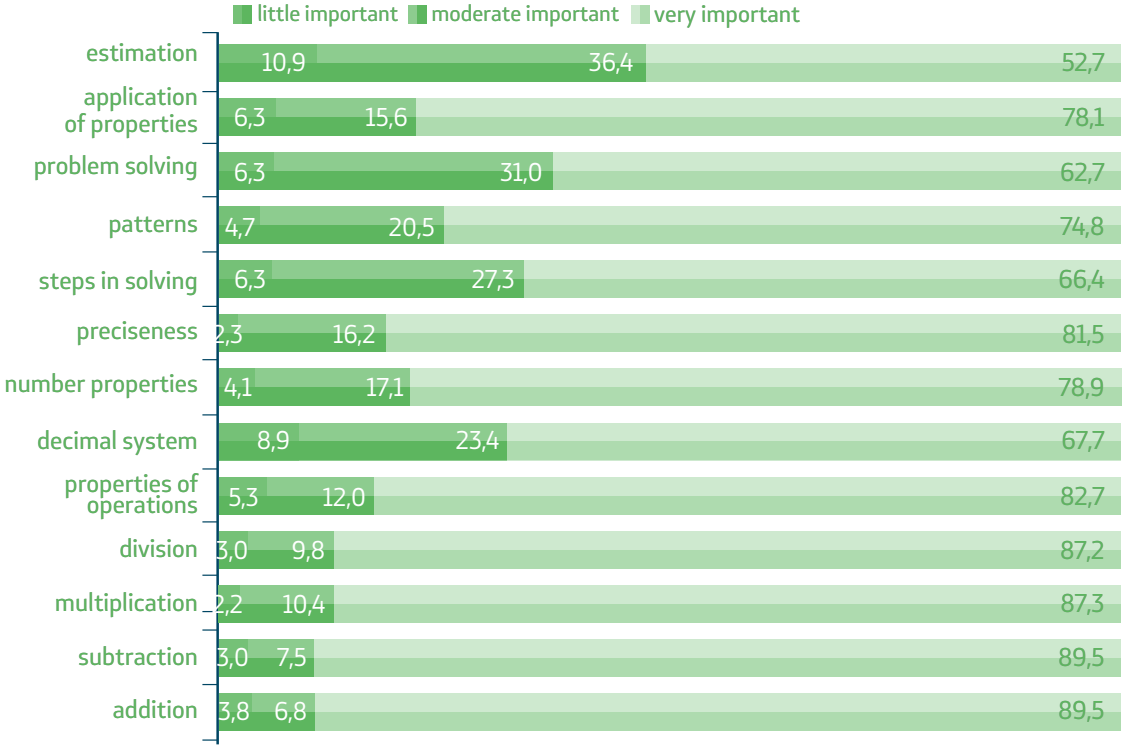
1.4.2. The importance of achieving particular objectives at the end of Grade 3

The instruction is lead, to a great extend, by the implicit curriculum performed by teachers, i.e. the way how he/she understood the objectives and the importance applied to them. The objectives to which teachers gave replies and their relationship with the Ten Principles are given in the following table.

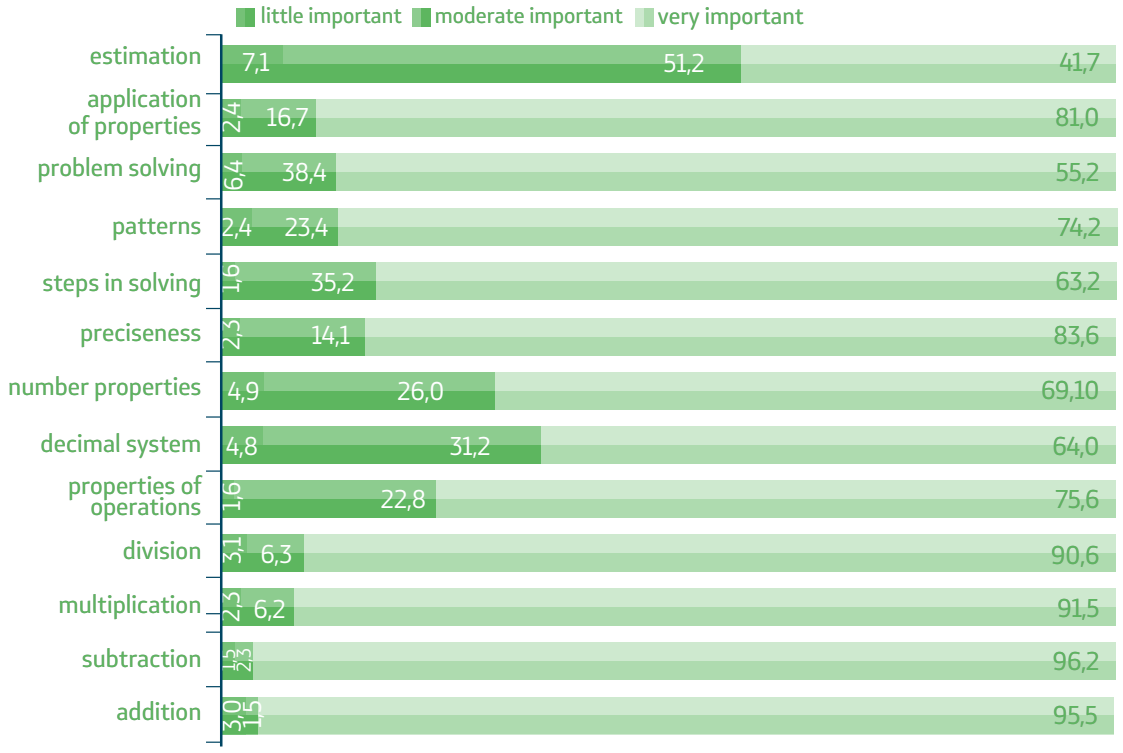
Table 6. *Relationships of objectives for students until the end of Grade 3 with the Ten Principles*

	Objective	Relationship with the Ten Principles
1	To understand addition and its importance	Balance between conceptual and procedural knowledge
2	To understand subtraction and its importance	Balance between conceptual and procedural knowledge
3	To understand multiplication and its importance	Balance between conceptual and procedural knowledge
4	To understand division and its importance	Balance between conceptual and procedural knowledge
5	To use the properties of operations	Procedural knowledge
6	To understand the decade system of numbers	Establishing the concept of number
7	To understand the basic properties of natural numbers	Establishing the concept of number
8	To be precise in counting	Procedural knowledge
9	To use usual procedures in doing numerical tasks	Procedural knowledge
10	To notice regularity in mathematics	Conceptual knowledge
11	To be able to solve a problem task regardless of the way they come to the solution	Using various strategies in solving
12	To use the four basic mathematical operations, the properties of operations to do the textual task	Procedural knowledge
13	To foresee the result of estimation	Conceptual knowledge

The importance (presented in percents) that teachers from project and non-project schools give to particular objectives in the instruction of mathematics is shown in the following two graphs.



Graph 5. *The importance that teachers from project schools give to particular objectives*



Graph 6. *The importance that teachers from non-project schools give to particular objectives*

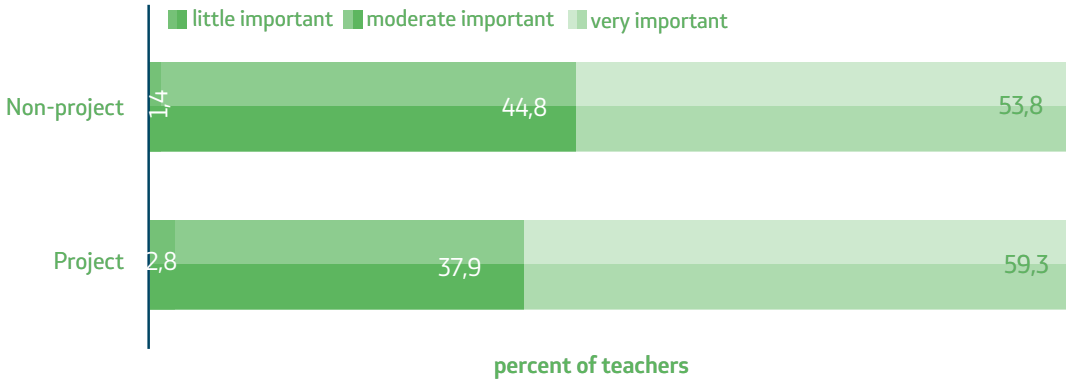
Although, the teachers consider that all of the mentioned mathematical knowledge/skills are important, particular attention is given to understanding the basic mathematical operations (addition, subtraction, multiplication and division). On the other side, less importance is given to the abilities for solving tasks using different strategies, as well as to foreseeing the result of the estimation. It means that they consider, as most important, the technical skill and the preciseness, while the creativity is seen as less important.

CONCLUSION

- There is no difference in the importance given to particular objectives by teacers fro project and non-project schools.
- The replies could point out to which objectives greater attention should be put on during the training and in the support given by teachers

1.4.3. Methodological freedom in curricula

In introducing innovations to the teaching methods, of particular importance is whether teachers consider that curricula enable them the needed freedom of selecting and using different methods. The opinions of the examined teachers are presented on the following graph.



Graph 7. *Opinion of teachers about methodological freedom in curricula*

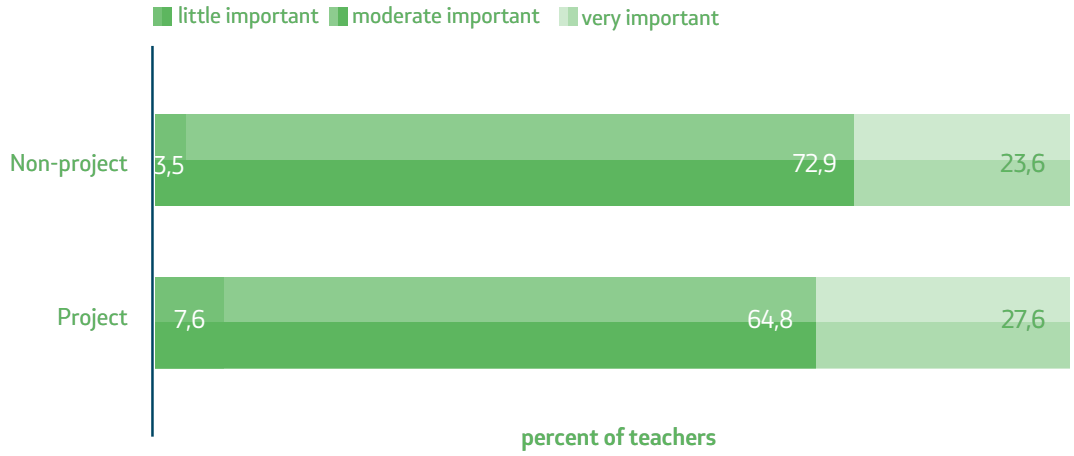
More than half of the teachers, consider, that the curriculum enables them to a great extent to choose the approach in teaching they consider to be adequate. however, about 40% of the teachers consider that the curriculum enables them to do it partially. there is no difference in the opinion between the teachers from project and from non-project schools.

CONCLUSION

- The majority of teachers consider that they have freedom concerning the methods in performing the instruction, but still there is a great number of teachers that fee that they do not have enough freedom of methods to use them in the most adequare way in the instruction of mathematics.
- In the course of the training they should be strenghened and be encouraged to use variety of methods and to undertake their own responsibility for the achievements of students.

1.4.4. Expectation of achievements higher than prescribed curricula outcomes

Due to the above explained level of requirements, we asked the teachers about their expectations of the students to achieve more than the expected outcomes in the curriculum. Their replies are shown in the following graph.



Graph 8. *Opinions of teachers whether their pupils could achieve more than expected outcomes prescribed by curriculum*

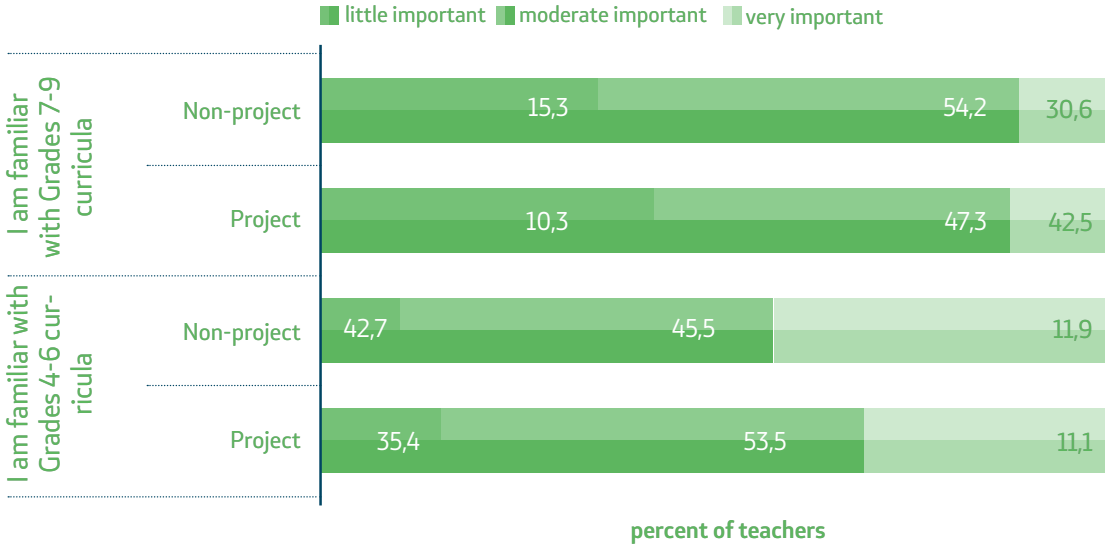
The majority of teachers (abouty 60%) from both project and non-project schools agree partially that students in their class could achieve more than that prescribed by the curriculum.

CONCLUSION

- Generally, the majority of teachers do not expect that their students could achieve more than that prescribed by the curriculum.
- There is no difference in the expectations of teachers from project and non-project schools.

1.4.5. Familiarity with the mathematics' curricula for the subsequent education cycles

Familiarity with the expected outcomes from learning mathematics until the end of primary education is considered to be an important asset in implementing the curricula in the first cycle. An indicator for the ongoing state in our schools, are the issues on familiarity with curricula for the subsequent education cycle. Teachers' responses are shown in the following graph.



Graph 9. *Familiarity with the curricula for subsequent cycles in primary education.*

About half of the teachers are partially familiar with the curricula for the subsequent cycles. However, while 31% of the teachers from project schools and 41% teachers from non-project schools are familiar to a great extent with the mathematics curricula for Grades 4-6, a smaller number (15% in project schools 10% in no-project schools) are not at all familiar with the mathematics curricula for Grades 7-9, a large number of the surveyed teachers (43% in project и 35% in non-project) are not at all familiar with them.

CONCLUSION

- Teachers from both project and non-project schools are not sufficiently familiar with the mathematics curricula for the subsequent cycles, and particularly with those in the last cycle (Grades 7-9).
- Difference in familiarity with curricula for the subsequent cycles of project and non-project schools and of teachers from project and non-project schools are not statistically significant.

I 2. TEACHERS' KNOWLEDGE OF MATHEMATICS

2.1. Achievements of teachers on the test in mathematics knowledge

METHOD OF MEASURING

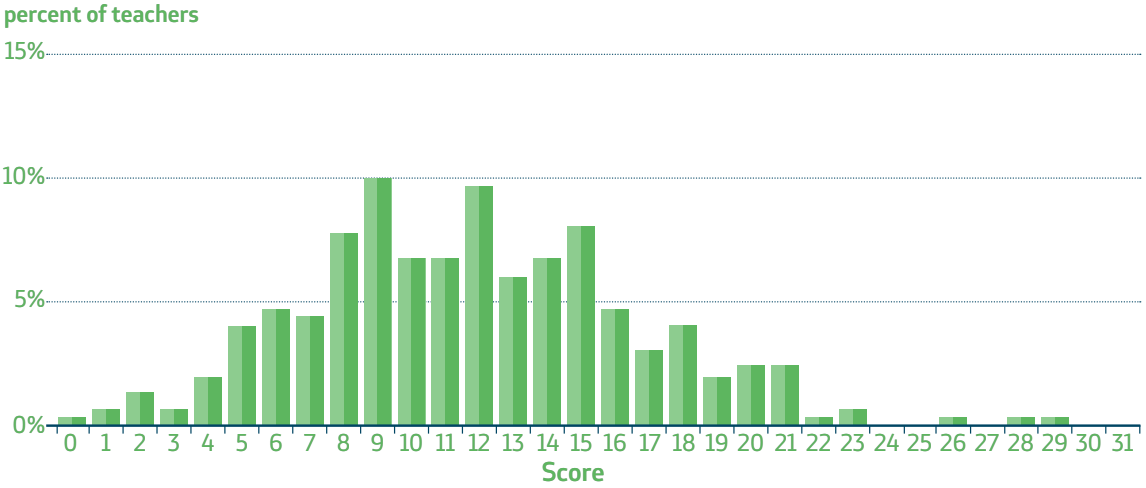
The test in teachers' mathematics knowledge has 15 tasks, out of which 8 are clusters with 3 or 4 items. We used these tasks to measure the knowledge and skills in these areas:

- Concept of number – 9 items;
- Operations and properties of operations – 19 items; and
- Problem situations – 6 items.

The test tasks, though designed for the teachers, from the aspect of mathematical content do not exceed the mathematics curricula for primary education, i.e. the expected results from students at the end of Grade 6.

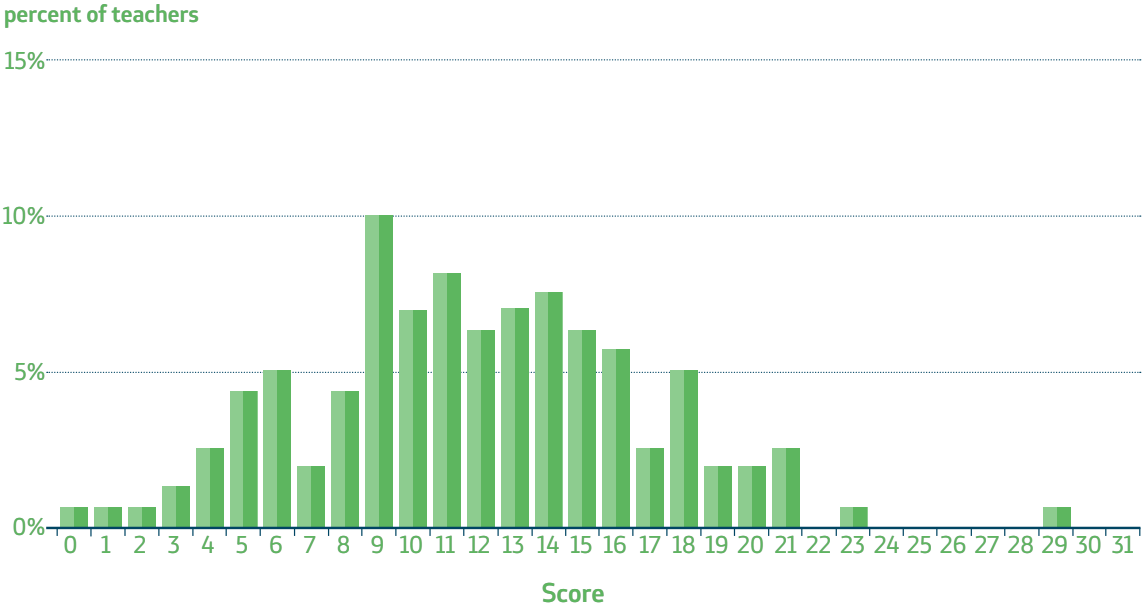
The limitations from the aspect of the small number of tasks (items) according to mathematics topics in the test, do not allow generalizing of conclusions for the entire topic. However, on the basis of the results, we could state the level of teachers' knowledge and skills which are explicitly measured by an adequate task in the test.

The average score at the test in mathematics of **all teachers** is 11,76 (maximum possible is 35), i.e. the average percent of correct answers is 33, 60%. The highest achieved score at the test is 29, and the largest is the percent of teachers (10,03%) who have a score of 9. The achievements at the test have a normal distribution, which is inclined more to the left side. It is characteristic that not a single teacher has achieved a score between 30 and 35.



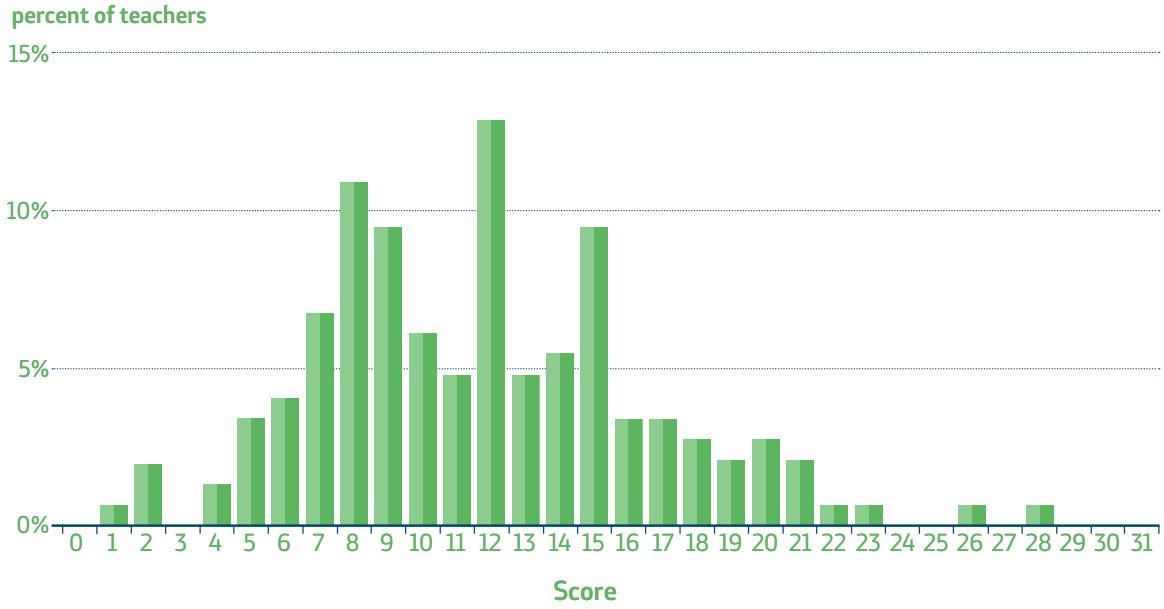
Graph 10. *Results of all teachers at the test in mathematics knowledge*

The average result at the test for teachers in project schools is 11.81, i.e. the average percent is 33, 75%. The highest achieved score is 29 by one teacher only, and the highest is the percent of teachers who have a score of 9. Out of 150 testing teachers, not a single one responded correcly to any of the requirements, and one teacher has a score of 1, i.e. a score of 2.



Graph 11. *Results of teachers from project schools at the test in mathematics knowledge*

The average score at the test with 149 tested teachers from non-project schools is 11.70 (out of the maximum of 33), i.e. the average percent is 33. 44%. One teacher only, achieved the highest result of 28, and the highest is the percent of teachers (12,75%) who have a score of 12.



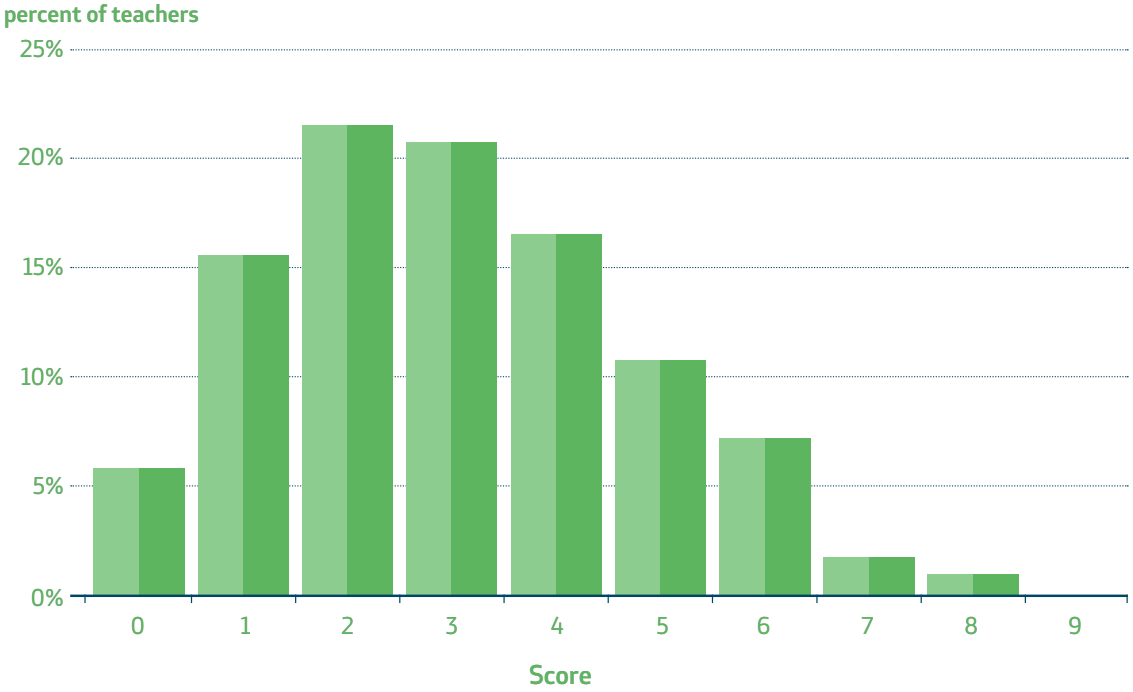
Graph 12. *Results of teachers from non-project schools at the test in mathematics knowledge*

CONCLUSION

- Teachers from both project and non-project schools do not differ in achievements at the test in mathematics.
- The achievements of teachers are lower than the expected ones.

2.1.2. Teachers achievement on Numbers

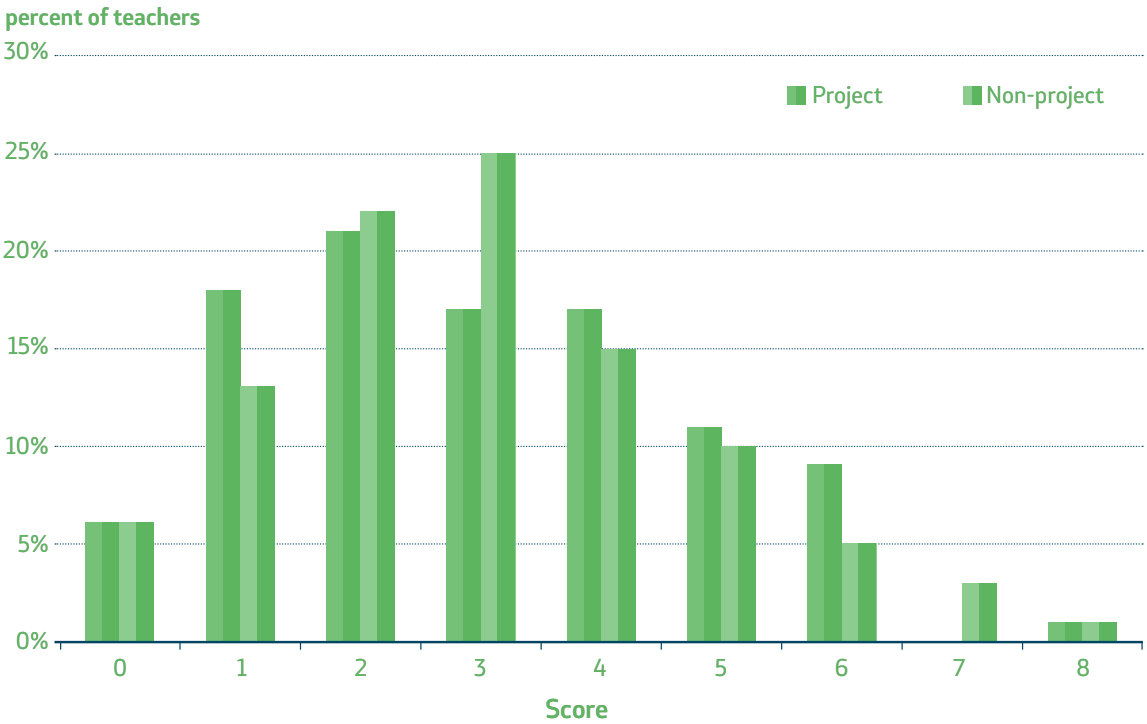
The average score of the nine items of the tasks on numbers of all teachers is 3.01 (maximum possible is 9), i.e. the average achieved percent is 33.48%. The highest achieved score at these items is 8, by three teachers only, and the highest is the percent of teachers (21,40%) who have a score of 2. Out of 299 tested teachers, 17 did not respond correctly to any of the items.



Graph 13. *Results of all teachers on Numbers*

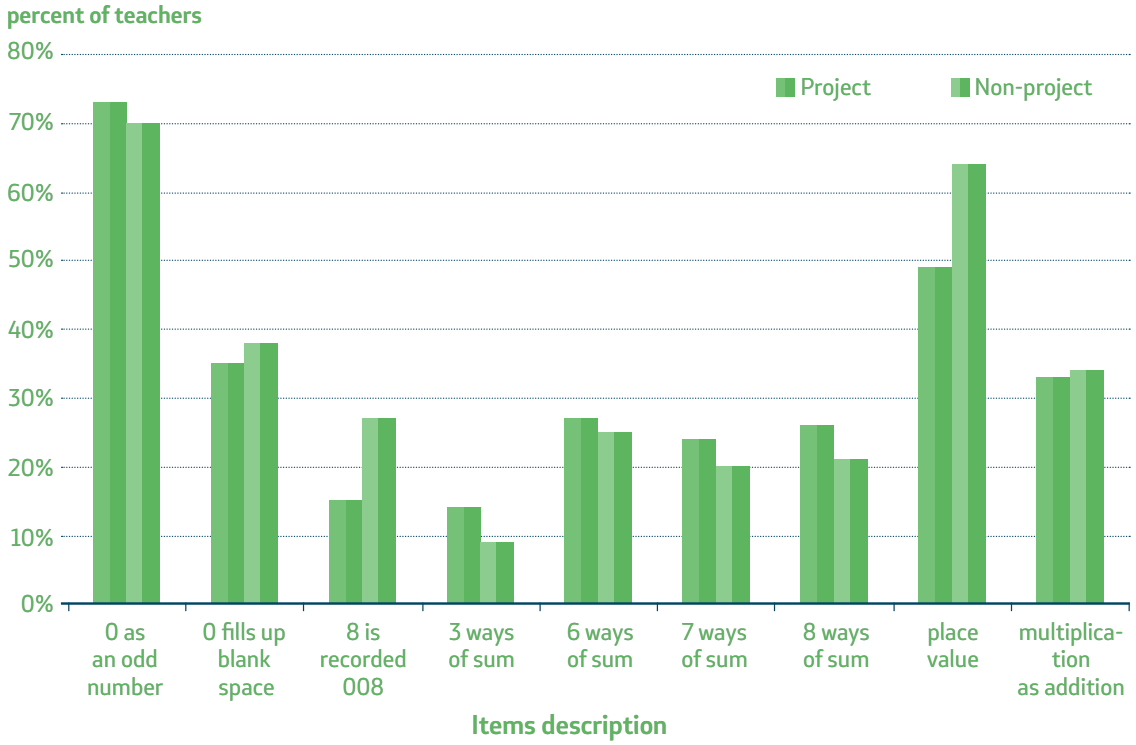
The average score of teachers **from project schools** is 2.97, i.e. the average percent is 32.96%. The highest achieved score is 8, by one teacher only, and the highest is the percent of teachers (20.67%) who have a score of 2. Out of 150 tested teachers, 9 did not respond to any of the items.

The average score of teachers **from non-project schools** is 3.06, and the average percent is 34%. Here, also, the highest achieved score is 8, the highest percent of teachers (24.83%) have a score of 3, and 8 did not respond correctly to any of the items.



Graph 14. *Results of teachers from project and non-project schools*

The graphic below shows the percents of correct responses given by teachers from project and from non-project schools, to each item in this topic.



Graph 15. *Achievements of teachers from project and non-project schools*

Specific are the responses of teachers related to the ways of recording a two-digit number as a sum of tenths and units.

Task

In the course of a training, the trainer gave the following problem to the teachers: *When we decompose the number 23, we usually think of tenths and units, so the number 23 we usually record as a sum of 2 tenths and 3 units. However, the number 23 could be recorded also as 23 units, or as a sum of 1 tenth and 13 units. In how many ways, the number 72 could be recorded as a sum of tenths and units?*

During the coffee break, several teachers were comparing and discussing their attempts to do the task. Below are given some different responses to the question of the trainer.

Encircle, in accordance with your opinion, EACH ONE of teachers' responses.

	Correct	Not correct	I am not sure
A) 3	1	2	3
B) 6	1	2	3
C) 7	1	2	3
D) 8	1	2	3

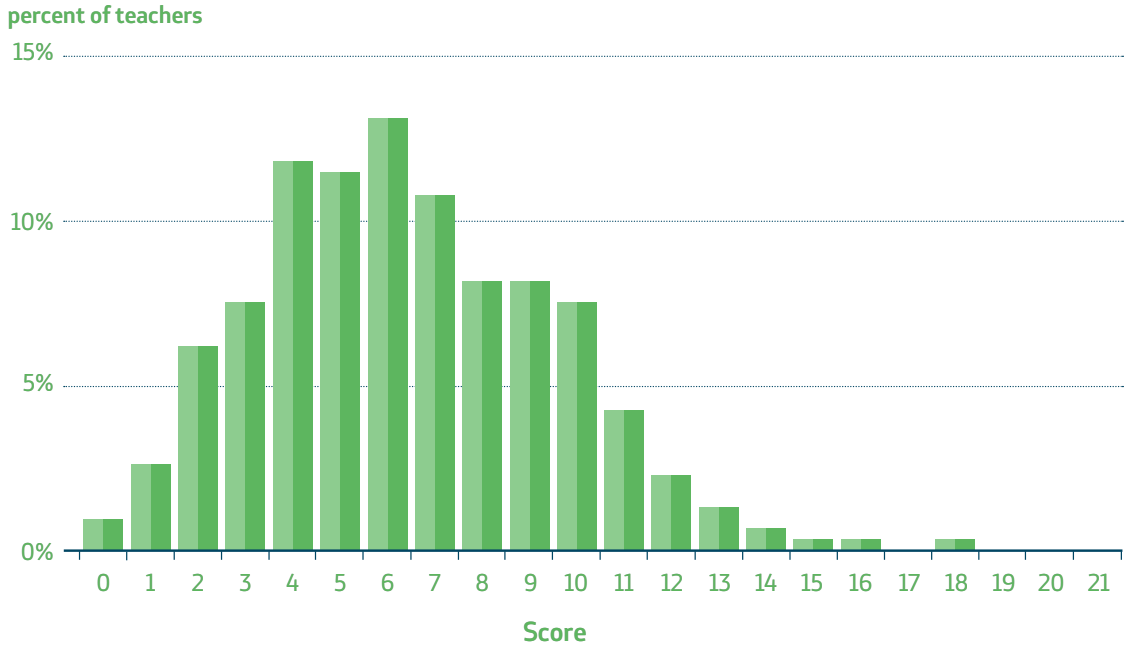
- The responses of teachers to this cluster task, show the way in which more than two thirds of teachers understand/do not understand the recording of numbers in the form of a sum;
- This implicitly denotes the way in which teachers teach their students about recording numbers in the form of a sum (that the correct recording is 7 tenths and 2 units);
- Namely, only 22% of teachers consider as correct the response that it could be recorded in 7 ways, taking into consideration also the recording of 0 tenths and 72 units.

CONCLUSION

- There is no statistically significant difference in the achievements of the tested teachers from project and from non-project schools at the tasks on numbers.
- The results are lower than the expected ones which could enable studying numbers in accordance with the directions of the project.

2.1.3. Achievements of teachers at the tasks on Operations and properties of operations

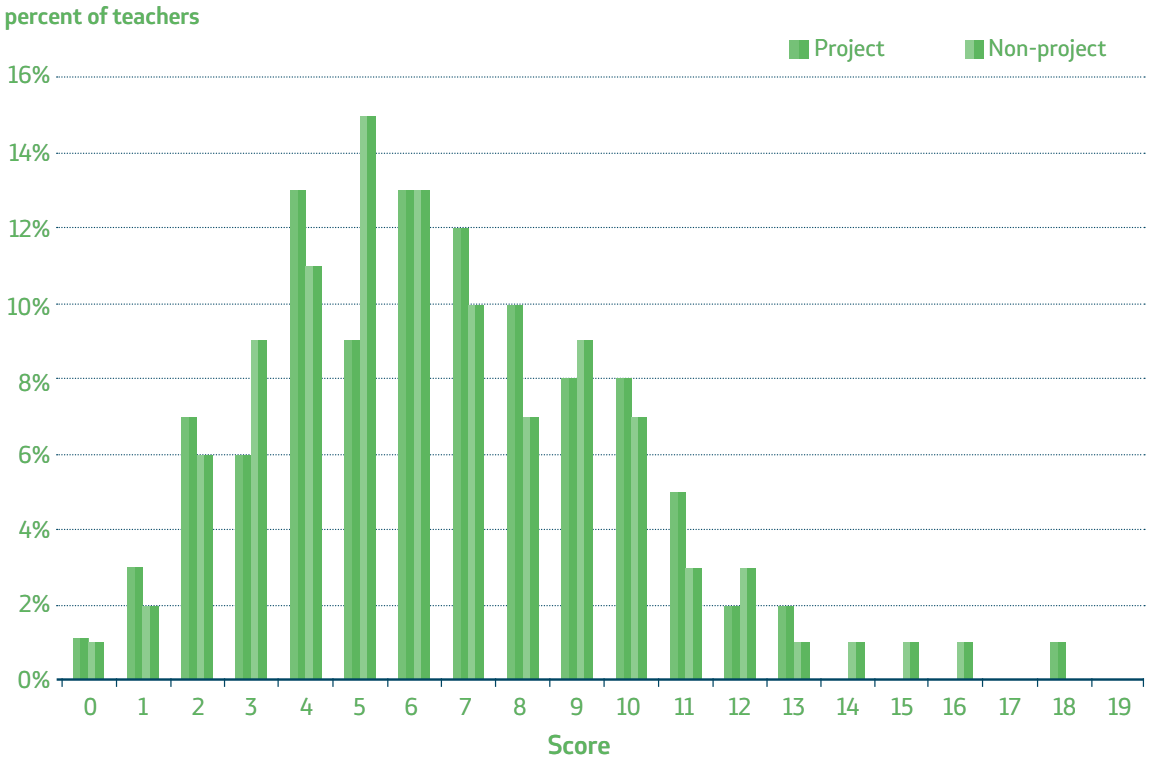
The average score at 21 items on operations and properties of operations of **all teachers** is 6.41 (maximum possible is 21), i.e. the average percent is 30.51%. The highest achieved score is 18, by one teacher only. The highest is the percent of teachers who have a score of 6. Out of 299 tested teachers, three did not give correct response to any of the items. The achievements in in this topic have a normal distribution, but it is more inclined to the left.



Graph 16. *Results of all teachers on Operations and properties of operations*

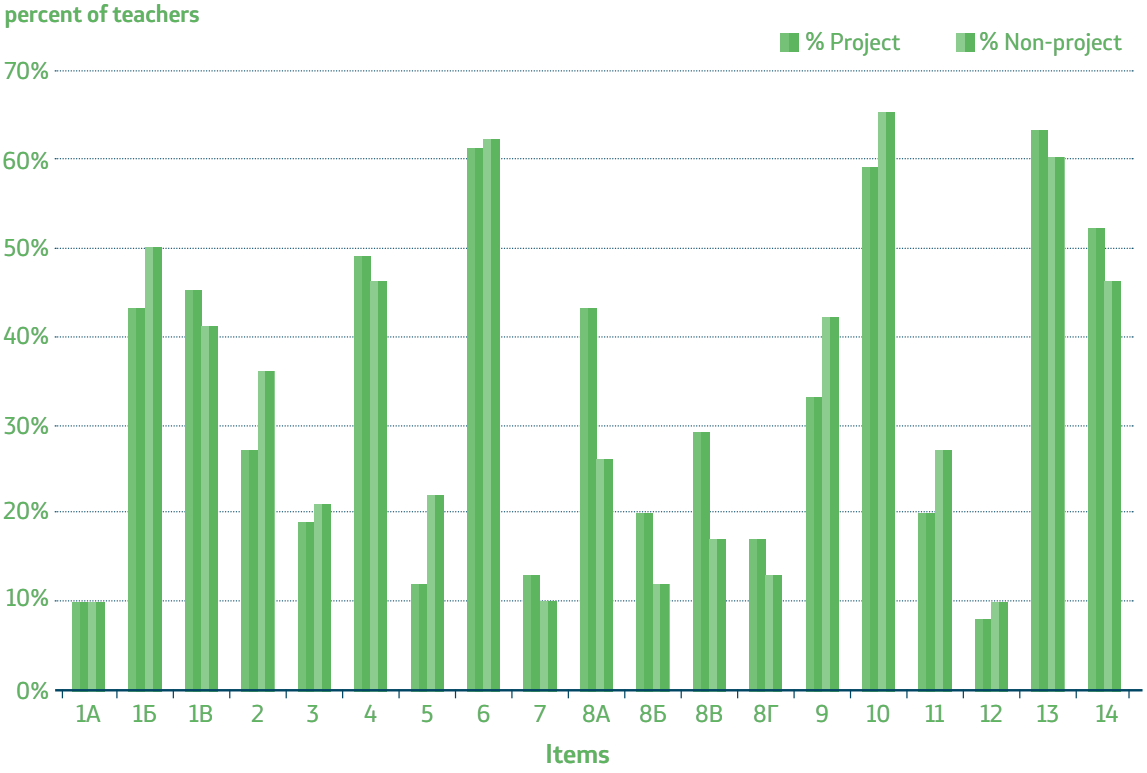
The average score of **teachers from project schools** is 6.45, i.e. the average achievement percent is 30.73%. The highest achieved score is 18, by one teacher only, and the highest is the percent of teachers (13.33%) who have a score of 6. Two teachers did not give correct response to neither of the tasks' items.

The average score of **teachers from non-project schools** is 6.36, i.e. the average percent of correct answers is 30.30%. The highest achieved score is 16, by one teacher only, and the highest is the percent of teachers who have a score of 6. Neither of the tested 149 teacher has not a single correct response. .



Graph 17. *Results of teachers from project and non-project schools*

The graph below shows the percents of the correct responses by teachers from project and from non-project schools on each item on operations and properties of operations.



Graph 18. *Achievements of teachers from project and non-project schools on operations and properties of operations items*

Table 7. *Description of items*

Item	Description
1A	Multiplication procedure A
1B	Multiplication procedure B
1C	Multiplication procedure C
2	Addition in Z
3	Subtraction in Z
4	Multiplication in Z
5	Division in Z
6	Subtraction up to 100
7	Addition in 6 ways
8A	Distributive property A
8B	Distributive property B
8C	Distributive property C
8D	Distributive property D
9	Application of commutative property
10	Application of associative property
11	Application of distributive property
12	Equation
13	Subtraction with borrowing
14	Estimation of sum

Characteristics are the responses on the task which requires reviewing various procedures in multiplication and their adequacy with either of the two natural numbers.

Task

magine that with your students you work on multiplication of two-digit numbers. You have noticed in students' notebooks the following ways of multiplication of numbers 35 and 25:

Student A	Student B	Student C
35 · 25 25 150 100 + 600 875	35 · 25 175 + 700 875	35 · 25 125 + 75 875

Which one of these students use the procedure that is adequate in multiplication of any of two natural numbers?

Encircle, due to your opinion for EACH ONE of the responses.

	Use the procedure which IS adequate for any two natural numbers	*use the procedure which IS NOT adequate for any two natural numbers*	I am not sure
Student A	1	2	3
Student B	1	2	3
Student C	1	2	3

Although each one of students A, B and C, uses a procedure that is adequate in multiplication of any two natural numbers, teachers' responses are only 10% for student A, 46% for students B and 43% for student C;

Worrying attitude is that even 46% of the teachers responded that the procedure of the student A, IS NOT adequate, and what this student did is the same as that done by student B, except that he recorded the way in which he/she thinks when doing multiplication.

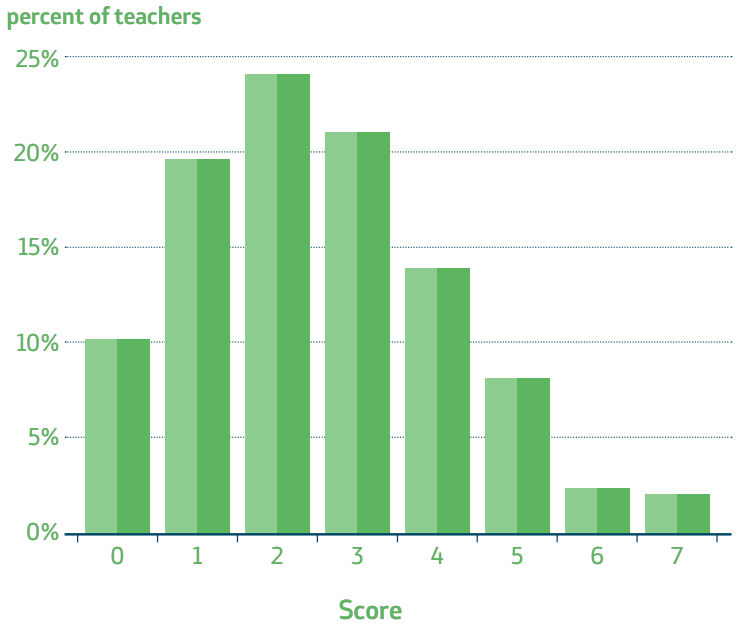
The largest is the percent of teachers who state that the procedure of student B is adequate, thinking of the term "procedure" i.e. as recording which is "most common", obviously most demanded or most highly valued by teachers.

CONCLUSION

- There is no statistically significant difference in the achievements of teachers from project and non-project schools on the tasks in this topic.
- The results in operations and properties of operations are lower than the expected ones which enable instruction on operations in accordance with the guidelines of the Project.

2.1.4. Achievement of teachers on Textual tasks and problems

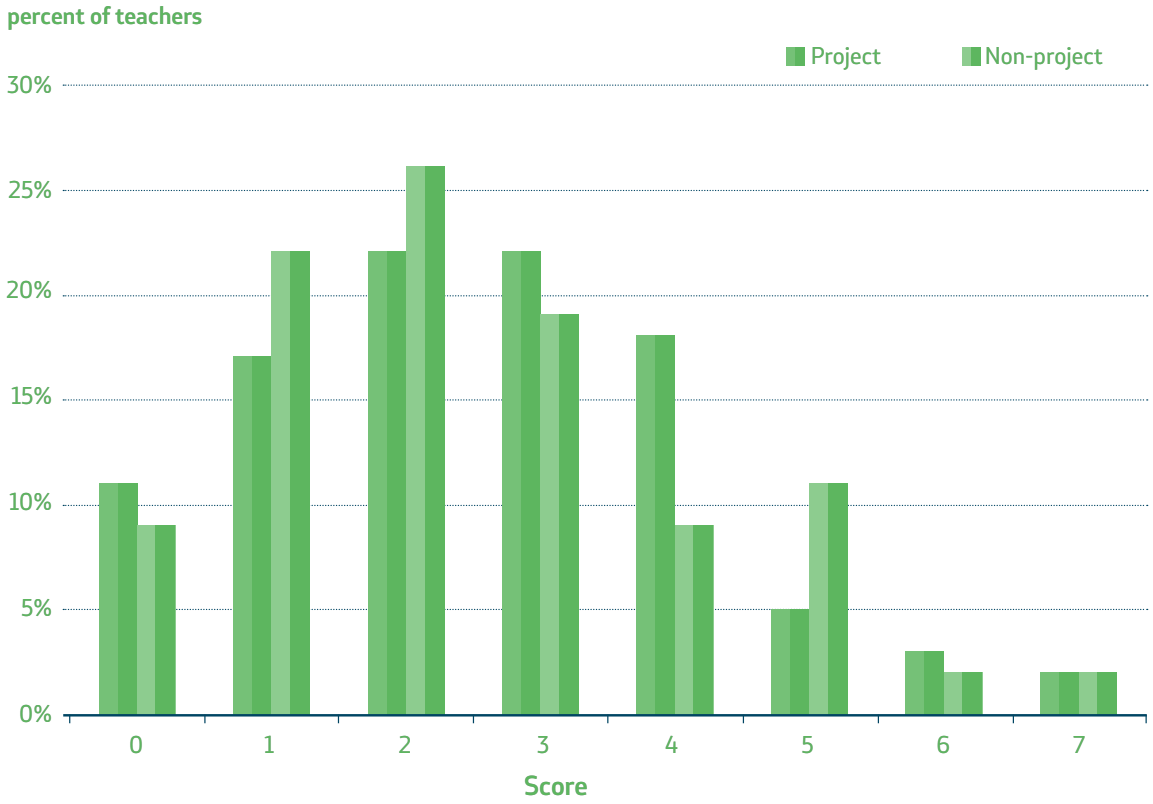
The average score on the seven items of the textual tasks and in solving mathematical problems of **all teachers** is 2.52 (maximum possible is 7), i.e. average percent is 36.02%. The maximum score is achieved by 6 teachers, and the largest is the percent of teachers (23.75%) who have a score of 2. Out of all tested teachers, 10% did not give correct answer to any of the items.



Graph 19. *Results of all teachers on Textual tasks and problems*

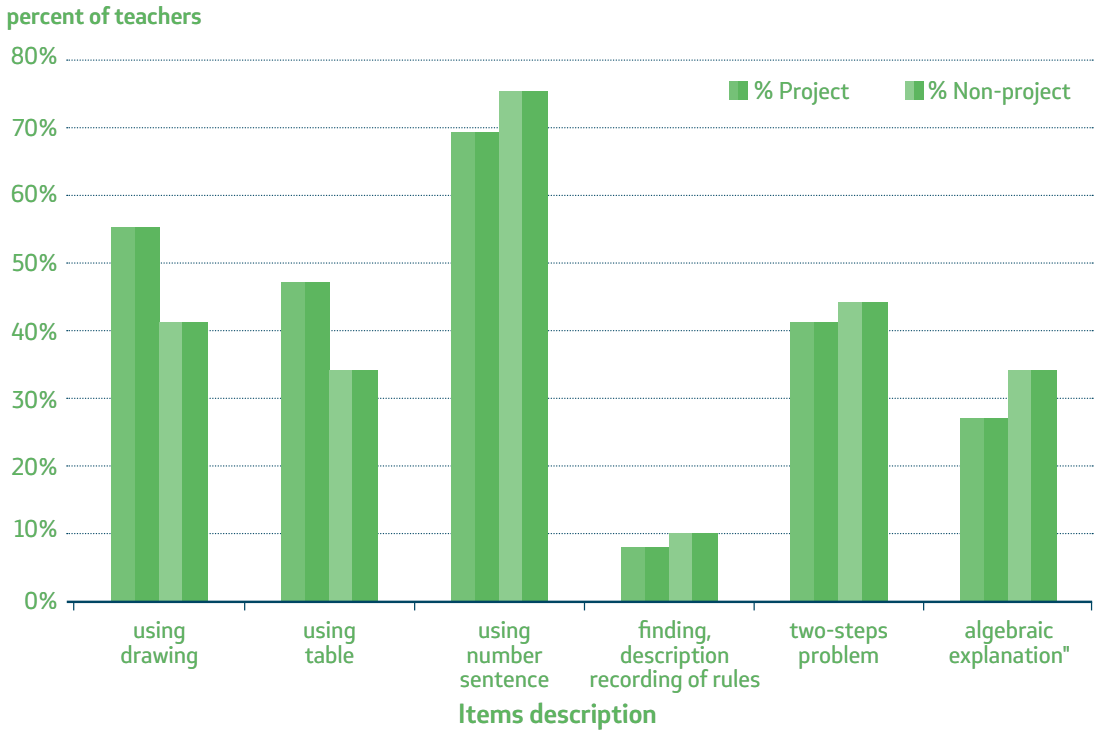
The average score of teachers from project schools is 2.56, i.e. average score of doing them well is 36.57%. The maximum score is achieved by three trachers, and one third of teachers have a score of 2, i.e. a score of 3. Out of the 150 tested teachers, 16 did not give correct response to any of the items.

The average score of teachers from non-project schools is 2.48, i.e. average percent is 35.47%. Three teachers have a maximum score, but 14 did not give correct response to any of the items.



Graph 20. *Results of teachers from project and non-project schools*

The graph below shows the percents of correct responses to each of the items, given by teachers from project and from non-project schools.



Graph 21. *Achievements of teachers from project and from non-project schools*

Though, the lowest results were achieved in this area at the task which requires from teachers to notice a given rule, to describe it in words and present it in an algebraic way[11], hew we would retain our attention to a cluster tasks in which a problem situation is given and which could be solved in several ways.

Task

Ana, the teacher, gave the following task to the advanced students A, B, and C:

Ivo, a Grade 7 student, sells used colored pencils. He sells 2 colored pencils for 3 denars. In a week, Ivo earned 48 denars. How many colored pencils have Ivo sold out?

In the table, three correct responses to the task were given, but solved in various methods: by using a drawing, by using a table and by creating and solving a number sentence. The teachers were required to give response to each method: to denote whether solution an answer it is completely correct, partially correct, not correct, or "I am not sure about the response".

- 75% of teachers consider that the solution by creating and solving a number sentence is completely correct;
- 34% of teachers consider that the solution by using a table is completely correct, but 14% of teachers consider that it IS NOT correct;
- 41% of teachers consider that the solution by using a drawing is correct, while 31% consider that this solution is partially correct.

Such a situation enables to suppose that the tested teachers give the least importance to the ability in solving the tasks in various ways and to the creativity in the used procedure, but they value highest, and probably use it in their teaching the arithmetic or algebraic way of solving textual mathematical tasks and problems. On the other side the Project promotes acquisition of skills in solving textual tasks and problem situations in various ways, and one of the objectives in the curriculum Grade 3 is "to enable students to notice problem situations in everyday life and to find ways in solving it".

CONCLUSION

- There is no statistically significant difference in the achievements of teachers from project and non-project schools.
- The results are lower than the expected ones which could enable teaching textual tasks and problems in accordance with the guidelines of the Project.

11 Similar tasks, but in a more complex form are present at the municipal competitions in primary education

I 3. SUPPORT TO CHANGES IN LEARNING MATHEMATICS

METHOD OF MEASURING

The support to changes in learning mathematics is considered to be an important factor to introducing changes in the instruction that derive from the training in Thinking Mathematics.

It was mainly examined by interviews with school principals and pedagogues/ psychologists from project schools. They were both given 13 questions that related to:

- satisfaction with achievements in mathematics in the grade teaching,
- expectations from students' achievements;
- activities for promoting the instruction of mathematics;
- cooperation of teachers concerning the teaching of mathematics;
- equipment for mathematics in the grade teaching cycle;
- experience from previous projects

The responses from the interviews are analyzed qualitatively.

In the questionnaire for teachers, there were also 2 questions related to the cooperation and the support in introducing innovations. The responses to these questions are shown graphically..

3.1. Satisfaction with achievements of students in mathematics

More than half of those interviewed (19, out of which 12 are school principals) expressed *satisfaction* with student's achievements in mathematics in the grade teaching cycle. For part of them, the high marks and the students' achievements at competitions are a good indicator for the performance. To a smaller number of them (mainly psychologists), the good results are due to using modern methods in teaching and in assessing, to which the teachers were trained.

Partial satisfaction was expressed by 6 of the interviewed. For the majority of them, the achievements are partially good because the curicula are overabundant and a large number of students are not able to master them.

No satisfaction was expressed by 9 of the interviewed. Dissatisfaction is due to various reasons. For part of them, it is a result of using inadequate teaching methods, for others it is because of lacking adequate teaching aids and equipment, and for some others it is because students do not engage themselves enough in learning mathematics. Two of them stated that students are not trained sufficiently to do textual tasks. The support from parents is not satisfactory, and this was mentioned as a reason for the unsatisfactory achievements.

Dissatisfaction with achievements is expressed most by the managing staff in the rural schools and by school pedagogues/psychologists.

3.2. Considerations about the possibility of improving the achievements

All of the interviewed consider that students' achievements *could be improved.* The majority of them, and especially the psychologists, believe that the *key* for improving the achievements is with *teachers*. They should arouse interest with students by using modern methods of instruction. They consider that such methods would arouse the interest for creative and logical thinking, which is important for mathematics.

> *Part of students could achieve more, but methods of instruction and the approach in work should be changed, and conditions for greater motivation of students and stimulus for their logical and creative thinking should be developed.* (psychologists)

Some of the interviewed pointed out that in order to accomplish higher achievements, financial support is needed, in addition to the training of teachers.

Small part of them, (mostly school principals), consider that the technical facilities are the key factor (available space for performing teaching, teaching aids, computerization, etc.).

> *I think that by introducing teaching in special clasrooms and computerization of the instruction, students could achieve better results.* (school principal)

Only a small part of the examinees, consider that the key factor for higher achievements is the aptitude of students for mathematics, as well as the support by parents.

CONCLUSION

- The satisfaction of the majority of the managing staff, mostly of school principals, with the achievements in mathematics in the grade teaching cycle, is not often based on sound indicators of success, as are those by objective measuring, comparisons with national standards or comparisons at international level. There is ground for schools to develop different culture of measuring own success which should be promoted within the framework of the Project Thinking Mathematics.
- On the other side, the considerations that the achievements could be improved primarily by additional training of teachers is a good ground for giving support to the project activities by the managing staff in project schools.

3.3. Support for improvement of mathematics teaching and learning

3.3.1. Activities for improving the instruction of mathematics undertaken by project schools

To the question: "Have you undertaken any activity for improving the instruction of mahematics, so far? What?" more than half of the interviewed (22, out of which 11 school principals) pointed out that they are trying to improve the teaching of mathematics by giving support to further professional training of teachers. They encourage the participation of teachers at workshops for acquiring modern teaching methods and their dissemination in the school, or they themselves suggest them to use new traching methods.

About less than a third of them (11) make efforts to encorage cooperation between teachers through support and advises, most often within the professional staff meetings. Part of them mentioned that they practice exchange of visits to classes. 11 from the interviewed mentioned that they are trying to improve the teaching of mathematics by providing new technical equipment (teaching aids, computers, LSD projector).

3.3.2. Cooperation of teachers related to the teaching of mathematics.

Mutual cooperation of teachers from the grade teaching cycle

Almost all of the interviewed (32) responded that grade teachers cooperate well, most often within the professional staff acting bodies, and out of them, in planning and in carrying out the teaching.

Grade teachers cooperate and help each other in all activities and duties (planning, exchange of experiences, making teaching aids). Professional acting bodies, also, are used as a form in giving help, for. ex. in presenting discusions ….. (school principal)

However, the school principal and the pedagogue from a school, pointed out that though the horizontal cooperation (between teachers teaching in the same grade) is good, that one between the teachers from different grades is weaker.

The professional acting body on one grade level, as a team, works well, they plan together, stay at school after classes. The cooperation betwen grades is feeble, for ex. Grade 4 does not cooperate with Grade 1, etc.(pedagogue)

Cooperation between grade teachers and subject teachers

The considerations of the pedagogues/psychologists related to the cooperation of the grade teachers with the subject teachers of mathematics are rather divided. Hoever, there dominates the opinion (expressed by 14 people) that such coperation exists, but it should be more frequent and и more deepened.

Cooperation could be improved. It exists, but is not continuous. Grade teachers should ask for help from subject teachers. Subject teacher asks the grade teacher why a certain student has problems in learning or knows a lot.(school principal)

11 from the interviewed consider that the cooperation is sufficient and qualitative, and they stress particularly the communication between teachers at the transition of students from Grade 4 to Grade 5.

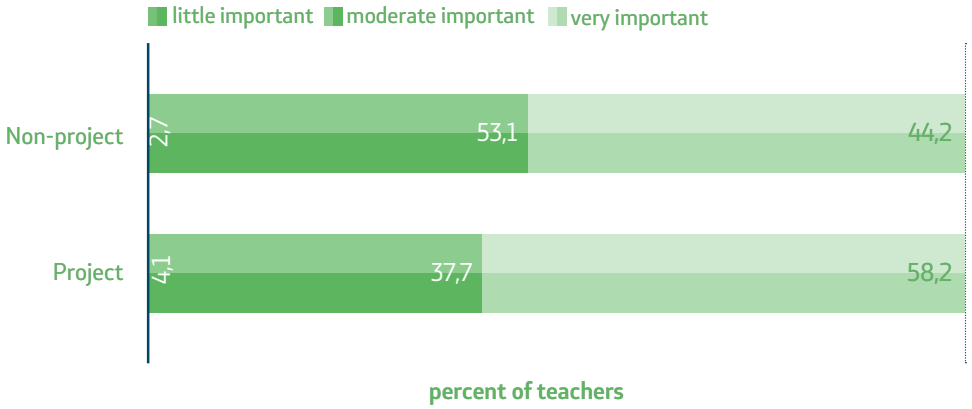
There is a particular cooperation when students move from Grade 4 to Grade 5, giving them own opinion about the students in that subject. (psychologist)

8 ouf the interviewed consider that the cooperation is insufficient, or that it does not exist. Part of them point out that the reason for that is the insufficient encouragement given by the management team for doing it, and that the school premises do not provide facility for meetings of grade teachers with subject teachers.

No, cooperation is minimal, because the school works in two shifts and the teachers lack opportunities to meet. The meetings are possible only at the staff meetings of the teachers' council, etc. (pedagogue)

3.3.3. Support to teachers

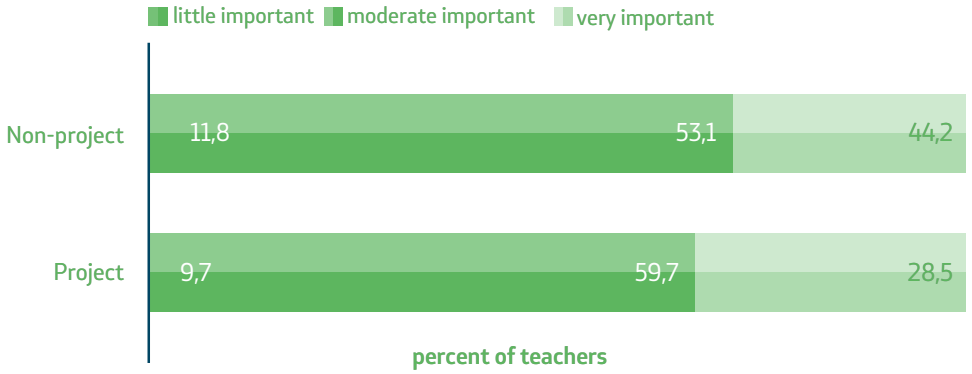
The grade teachers, also, to the question in the questionnaire, responded to what extent they cooperate with other teachers related to the teaching of mathematics. Their responses are given in the graph below.



Graph 22. *To what extent teachers cooperate with other teachers*

Teachers from project schools, more than those from the non-project schools, cooperate to a great extent with other teachers on issues related to the teaching of mathematics.

Teachers were asked whether they could rely in support at the school in introducing innovations in the instruction.



Graph 23. *Sense of support for changes*

About 1/3 of teachers from project and non-project schools consider that they could get support, to a great extent, for changes in the teaching of mathematics, and about 10% consider that they would not be supported in introducing changes.

CONCLUSION

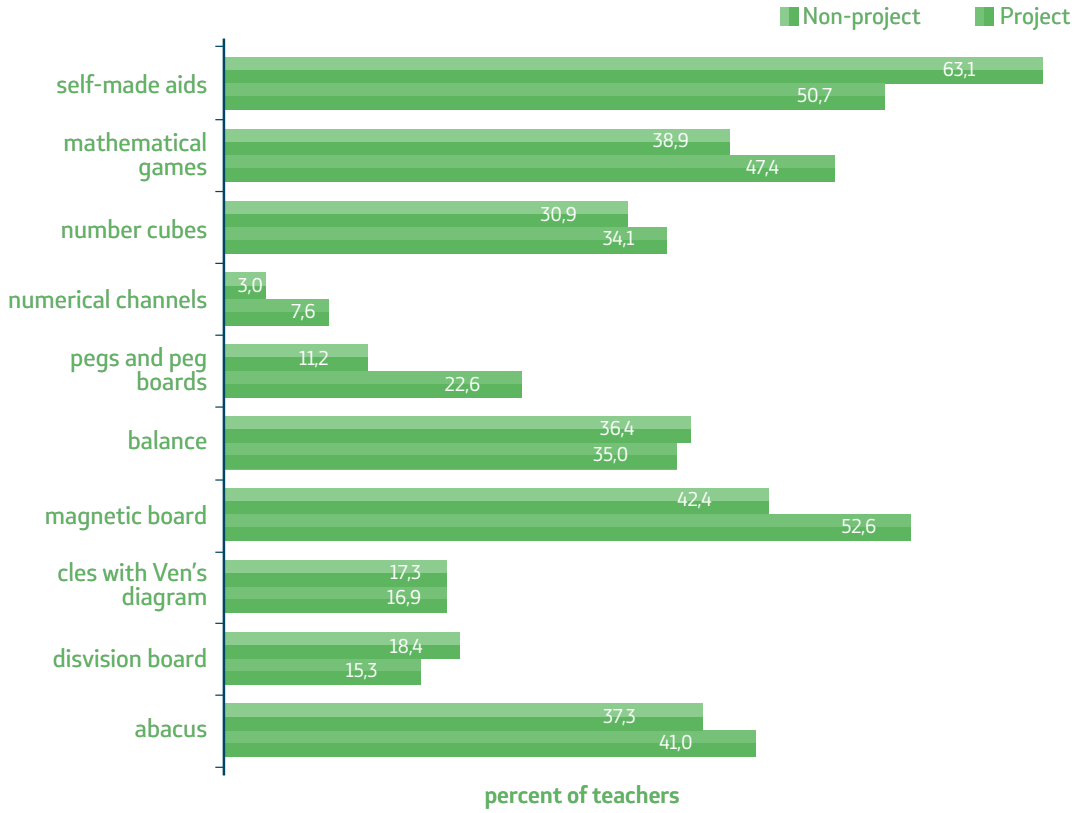
The practiced approaches in improving the teaching so far, were the support to the professional development of teachers through additional training and mutual cooperation. The majority of the teachers consider that they cooperate well with colleagues, and they could count on the support from the school in introducing innovations in the teaching of mathematics. It is a good base for introducing project activities. However, the cooperation of grade teachers with the subject teachers is not adequate, and is necessary, particularly when you take into consideration the data about the insufficient familiarity with the mathematics curricula until the end of primary education (pages 26 and 27 in this Report) and the limited mathematics knowledge of the examined grade teachers (pages.27-36 in this Report).

3.4. Equipment of schools for the instruction of mathematics

One of the Ten Principles in *Thinking Mathematics* is the use of manipulative aids. part of them could be handy, made by teachers and students, but for some content purposely produced teaching aids are necessary. It is essential to provide abundance of various teaching aids.

Findings from the interviews with the school management staff show that part of school directors and pedagogues/psychologists are not informed about the teaching aids that are used in the instruction of mathematics. Out of those that responded that they are informed, the majority (13) stated that the school possesses only basic teaching aids, rulers, compasses, geometrical figures). The largest part of them are not needed for the grade teaching cycle. A small number (6) pointed out that they have moder teaching aids (scales, jigsaws), but lack magnetic boards, while 3 examines mentioned that in addition to the basic teaching aids they have magnetic boards also. испитаници кажале дека покрај основните средства имаат и магнетни табли. Only 5 of the interview pointed out that the school posses modern teaching aids (for ex. tulkid, pipin, electronic board, scales) etc. It is worrying that 3 of the examined stated that the school does not have any teaching aids for mathematics, but have only blackboard and chalk.

According to the responses of teachers concerning the teaching aids for the instruction of mathematics that they have in their classrooms, it could be concluded that the majority of schools are poorly equiped. In half of the classrooms in the project schools, and in 63% of the classrooms in the non-project schools there are teaching aids made by the teachers themselves, and the number of commercially produced teaching aids are found in a small number of classrooms. In addition to the teaching aids made by the teachers themselves, the project and non-project schools differ in te equipment with peg boards



Graph 24. *Equipment of classrooms with teaching aids*

CONCLUSION

- The equipment of classrooms with manipulative aids is not satisfactory. It can not to a satisfactory extend support the instruction in Thinking Mathematics.
- The managing staff is not sufficiently informed about the state with teaching aids, and it seems that they lack professional knowledge what is needed for the instruction of mathematics in the grade teaching cycle.

3.5. Preparedness of the managing staff to give support to the implementation of new approaches in the teaching of mathematics

The preparedness of the managing staff concerning the implementation of new approaches in the teaching of mathematics, and especially in *Thinking Mathematics* was examined through their opinion and experiences from other projects, and the gathered information and preparedness to give support to the activities in the Project *Thinking Mathematics* .

3.5.1. Involvement of schools in projects and experiences from the projects

The majority of the interviewed (21) consider that their school is, to a sufficient level, involved in projects, i.e. to an optimum level taking into consideration the posssibilites of the school and of the staff. 5 consider that there too many projects, and that for some schools it is impossible to pay sufficient attention to the requirements of each of the projects.

> *Yes, there are too many, because each project has its own requirements to which they should respond, prepare lists, forms, reports, which mainly go through the pedagogue. So, very little attention could be devoted truly to the work of each individual project.* (pedagogue)

Convincing is, that the majority of the managing staff (24) judge their experience in the projects as positive and satisfactory.

> *Positive experience: they helped in raising the quality, motivation of teachers and students, the cooperative and the positive climate in the school.* (school principal)

Only one pedagogue had negative experience because the projects were exhausting and insufficiently motivational, while two of the interviewed psychologists had combined perceptions related to the projects. One of them stated that though the projects represent an opportunity for further profesional training, the main problems are in:

The insufficient evaluation (professional, moral, material) of the efforts in the activities of the project by the majority of the project managers and those responsible in the schools, and by the educational authorities for promotion of education.

and

Lacking clear, particular and coordiated national program for promotion of education, and due to that with the adjusting of various projects.

3.5.2. Support to activities in Thinking Mathematics

For the sucess of each inovation that is introduced at school level, it is necessary, from the very start, to inform well all those involved in it including the managing staff about the goals and the strategies and to share same vision.

According to the responses from the interviews, abot half those interviewed (17) cstated that the have sufficient information abou the project *Thinking Mathemat-*

ics, but the majority of them were not able to give precise information about how much they about the Project. They were informed about the project in various ways.

Some of them were infored about it by their school teachers involved in the Project after the training, two of them mentioned that they have observed classes implementing the new methods.

Four of the interviewed (2 pedagogues 2 school principal) responded that they are not at all informed, while 8 have just basic information.

> *I know that it deals with raising the quality of instruction in mathematics using various teaching techniques.* (school principal)

> *I know that the aim is to train teachers for a better quality teaching of the subject, and together with it to improve the achievements of students in mathematics.* (psychologist)

Related to the issue how much the involvement into the Project was based on individual decision of individual teachers, and to what extent it is part of the school policy in promoting the teaching of mathematics, the situation is different in particular schools.

14, out of the interviewed, stated that the teachers taking part in the Project were involved on the basis of agrement with the school principal/the schol management staff, while according to, this decision was made individually on self innitiative by the teachers. The others were not informed about the way of involving the teachers others.

School pedagogues and psychologists most often se their role in the Project as giving support to the implementation of the activities, monitoring gthe activitiesand motivating the teachers, while the school principals put stress more on the logistic support and the managerial role in the Project.

Concerning the subsequent activities in the Project, about half of the interviewed stated that they have already planned the next activities within the framework of the Project. Some do not have a concrete plan when they would be included, while others have not defined the timeline yet. One third (13) do not have planned activities, and some others are not informed what would happen next with the Project.

CONCLUSION

- In half of the project schools, the involvement into the Project *Thinking Mathematics* is in agreement with the school managing staff, while in the other half it is on the initiative of individual teacher/s and it was not part of the school's plans and strategies. The managing staff, prior to starting the project activities in the school is not completely informed, or is completely uninformed about its goals and activnities that are to be implemented. Nevertheless, generally, there is openness with the managing staff in the project schools to give support to new projects and approaches in the instruction including those of UNICEF Project *Thinking Mathematics*. They see their role as supporters in the process of further professional training of teachers and in providing conditions for carrying out the project activities.
- However, in addition to the generally positive attitude to the project *Thinking Mathematics*, more detaled informaion should be provided to all those that would be included (the management staff, teachers, both from grade teaching and subject teaching cycles, teaching mathematics), with its goals and objectives prior to starting the project activities,that are necessary for successful implementation and planning of the training, giving support, monitoring and evaluation of the project activities at in-school level.

I 4. ACHIEVEMENTS OF STUDENTS

METHOD OF MEASURING

We base the estimation of the achievements of students upon the achieved scores in doing the tasks that measure conceptual and procedural knowledge, understanding and application of natural numbers, the four basic operations and their properties, as well as in solving textual tasks and problems

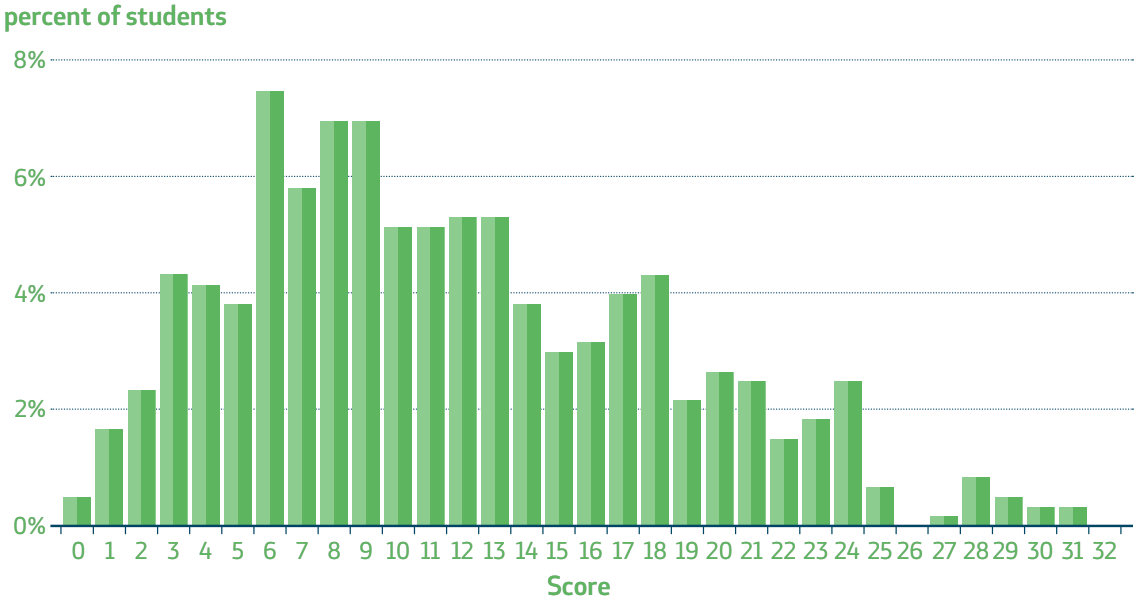
The test for students consisted of 19 tasks (21 items) which measure knowledge and skills in the areas of:

- Concept on number – 5 items;
- Operations and properties of operations – 11 items, and
- Problem situations – 5 items.

The limitation from the aspect of the small number of tasks (items) in the mentioned areas of the test, do not allow making generalizations of the conclusions for the entire area. Hence, the conclusions refer to the knowledge and skills that are explicitely measured by the tasks in the test.

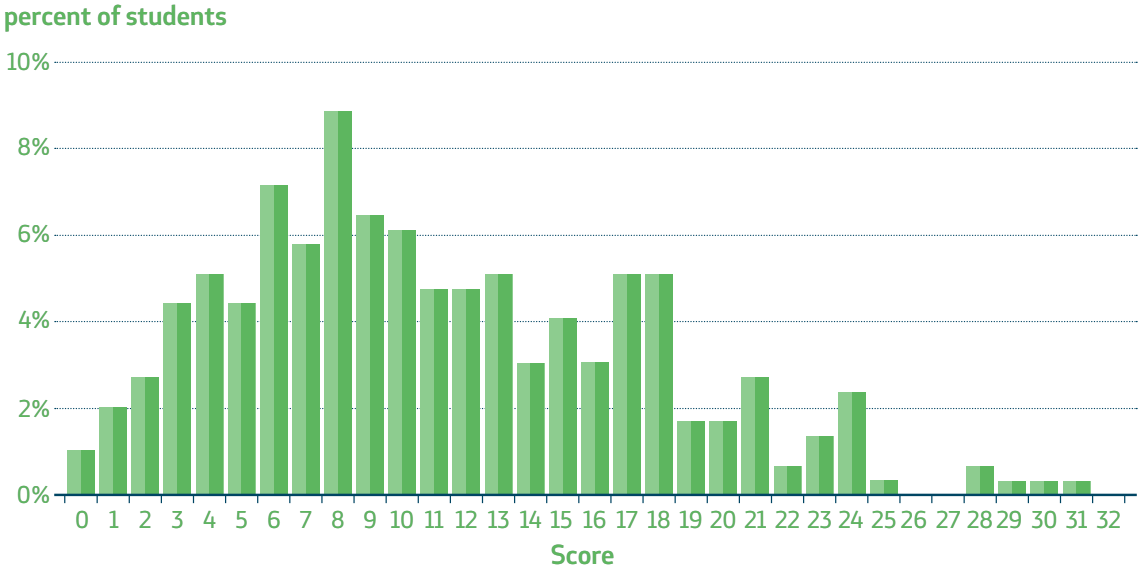
4.1. Achievements of students at the test in mathematics

The average score of **all students** at the test in mathematics is 11.64 (the maximum possible is 33), i.e the average percent correct answers is 35.29%. The maximum acieved score at the test is 31 by 2 students (0.34%), and the largest is the percent of students (7.54%) who have a score of 6. Oiut of the total number of tested students (597), 3 students did not correct response to any of the tasks. The achievements at the test have normal distribution inclined more to the left.



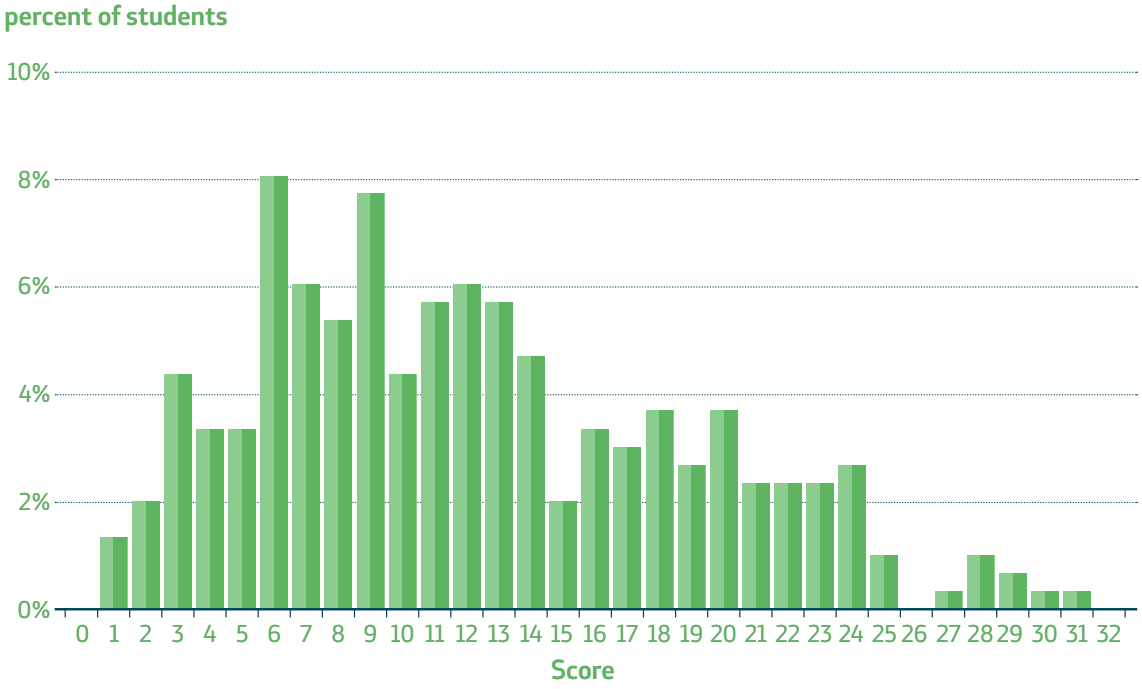
Graph 25. *Results of all students at the test in mathematics*

The average score of **students from project schools** at the test tasks is 11.10 , i.e. the average percent is 33.65%. The highest achieved score is 31 only by 1 student (0.33%), and the largest is the percent of students (8.7%) who have a score of 8. Out of the total number of tested students (299), 3 students did not give correct response to any of the items.



Graph 26. *Results of students from project schools at the test in mathematics*

The average score of **students from non-project schools** at the test is 12.19 out of the maximum of 33, i.e. the average percent is 36.93%. The highest achieved score is 31, by 1 student (0.34%), and the highest is the percent of students (7.7%) who have a score of 9. Out of the total number of tested students (298), all gave at least one correct response to the tasks and no student has done well all the tasks



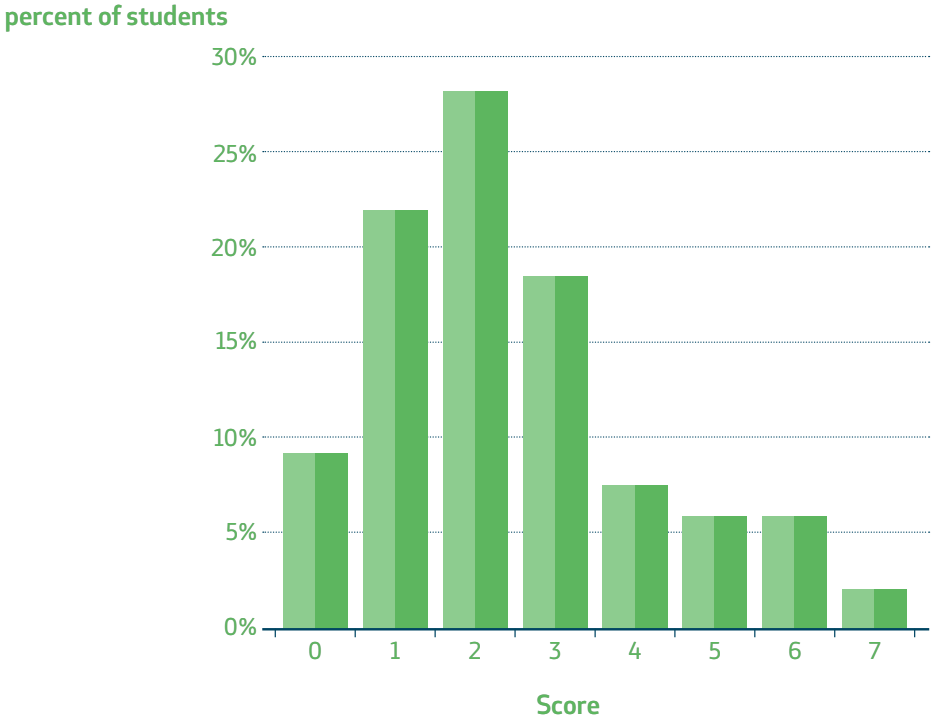
Graph 27. *Results of students from non-project schools at the test in mathematics*

CONCLUSION

- The results from the test are lower than the expected ones from those prescribed by the curiculum for Grade 3.
- Students from project and from non-project scghools do not differ in the achievements on the test in mathematics.

4.1.1. Students achievements on Number sense

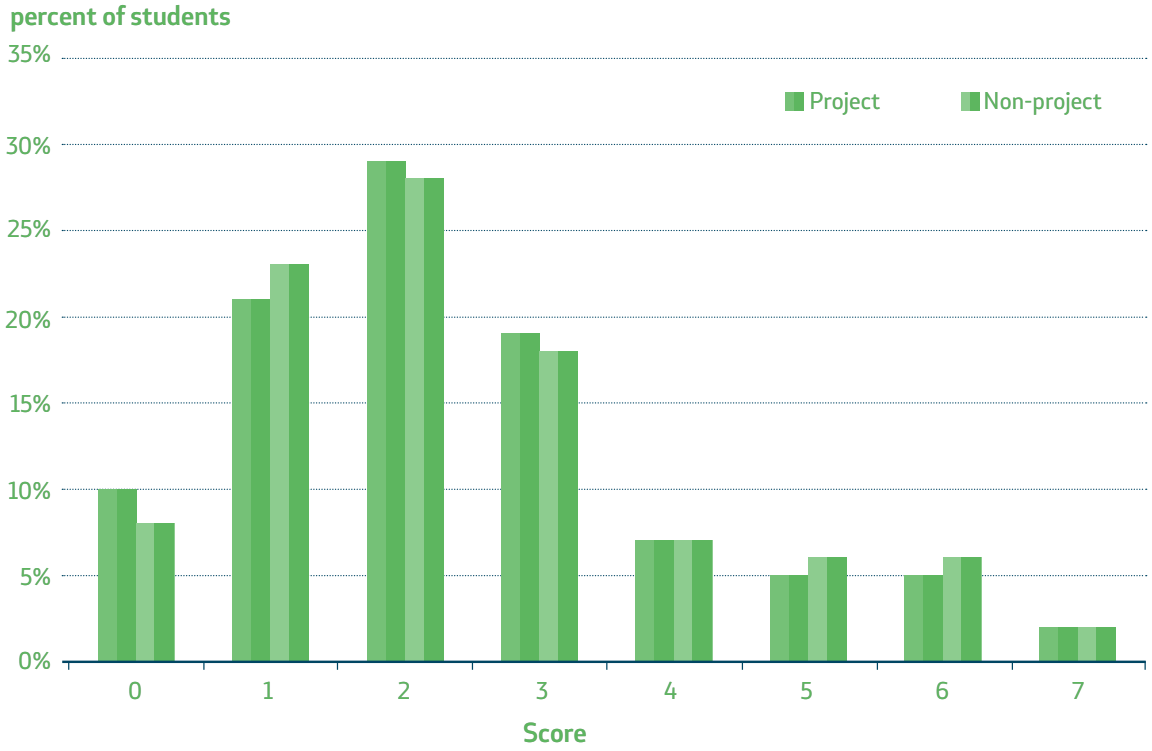
The average score of **all students** on the tasks related to numbers is 2.46 (maximum possible is 7), i.e. the average percent is 35.13%. The maximum scorre in this area is achieved by 14 students (2.35%), and the largest is the percent of students (28.48) who have a score of 2. Out of the total number of (597) tested students 9.21% did not give correct responses to any of the tasks. The achievements in the test have a normal distribution which is more inclined to the left.



Graph 28. *Results of students on Number sense*

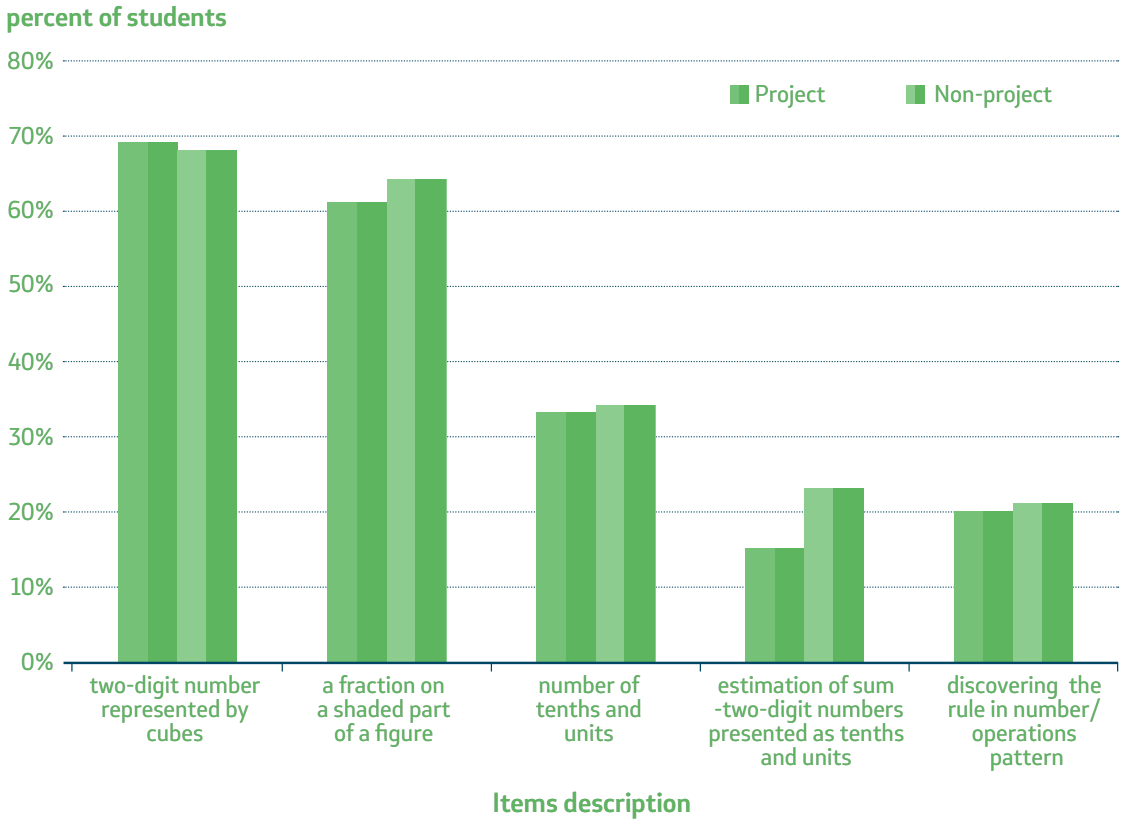
The average score of **students from project schools** on the tasks related numbers is 2.39, i.e the average percent is 34.21%. The maximum score is achieved by 6 students (2.01%), and the highest is the percent of students (29.10%) who have a score of 2. Out of the total number of tested students (299) 10.37% did not give positive response to any of the items.

The average score of ***students from non-project schools*** on the tasks related to numbers is 2.52, i.e. the average percent of correct answers is 36.05%. The maximum score of students from non-project schools in this area is achieved by 8 students (2.68%). In the non-project schools, also, the highest is the ercent of students (27.85%) who have a score of 2. Out of the total number of students (298), 8.05% did not give correct response to any of the items.



Graph 29. *Results of students from project and non-project schools*

The graph below shows the percents of correct responses of students from project and non-project schools on each item in numbers given in the test.



Graph 30. *Achievement of students from project and non-project schools according to tasks*

Specific are the responses of students to the requirement to find out the number of units in a given two-digit number.

Task

The number 35 has 2 tenths and ? units.

Which number should be recorded on the space before the question mark in order to get a correct answer?

(Encircle one correct response)

A. 5 units
B. 15 units
C. 25 units
D. 35 units

The responses of students on this task show the way how they "have been taught".

Namely, 34% students that 15 should be recorded in the square. But, 43% responded that 5 should be recorded. This leads to a supposition that:

- In most cases, when the task refers to presenting or describing numbers as a sum, the students are required and are expected to demonstrate one way, only, in presenting first all the tenthsн and then the remaining units, and

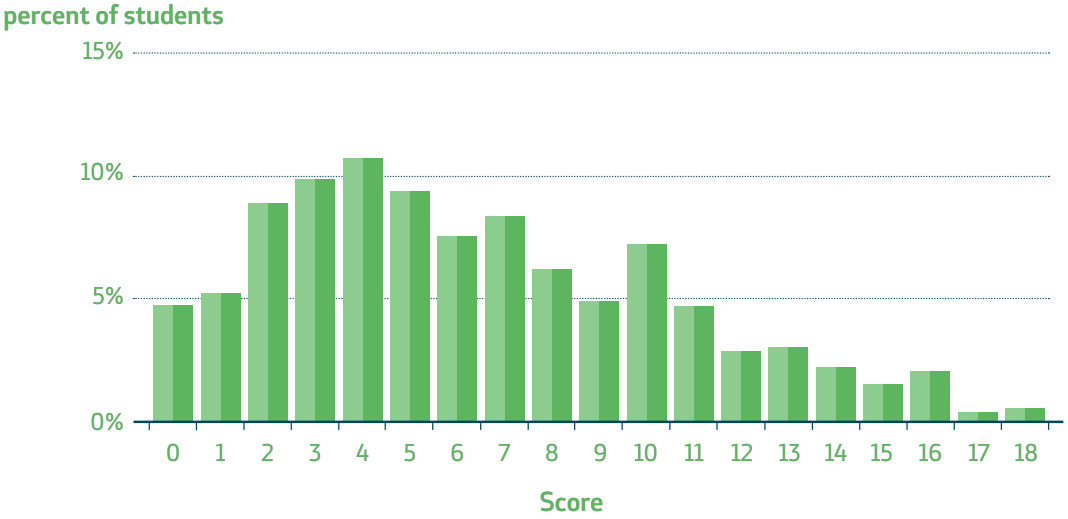
- In introducing the concept of number, the manipulative aids in composing, presenting, and decomposing of numbers..

CONCLUSION

- The results on tasks with numbers are lower than the expected results prescribed by the curriculum for Grade 3.
- There is no statistically significant difference in the achievements of students from project and non-project schools.

4.1.2. Students achievements at the tasks on Operations and their properties

The average score of **all students** on the tasks used to measure knowledge and skills on operations and properties of operations is 6.37 (maximum possible is 18), i.e. the average percent of correct answers is 35.36%. The maximum score 18 in this area has a frequency 3 (0.5%), and the largest is the percent of students (10.72%) who have a score 4. Out of (597) tested students 4.69% did not give correct response to any of the items. The achievements on the items have normal distribution which is more inclined to the left.

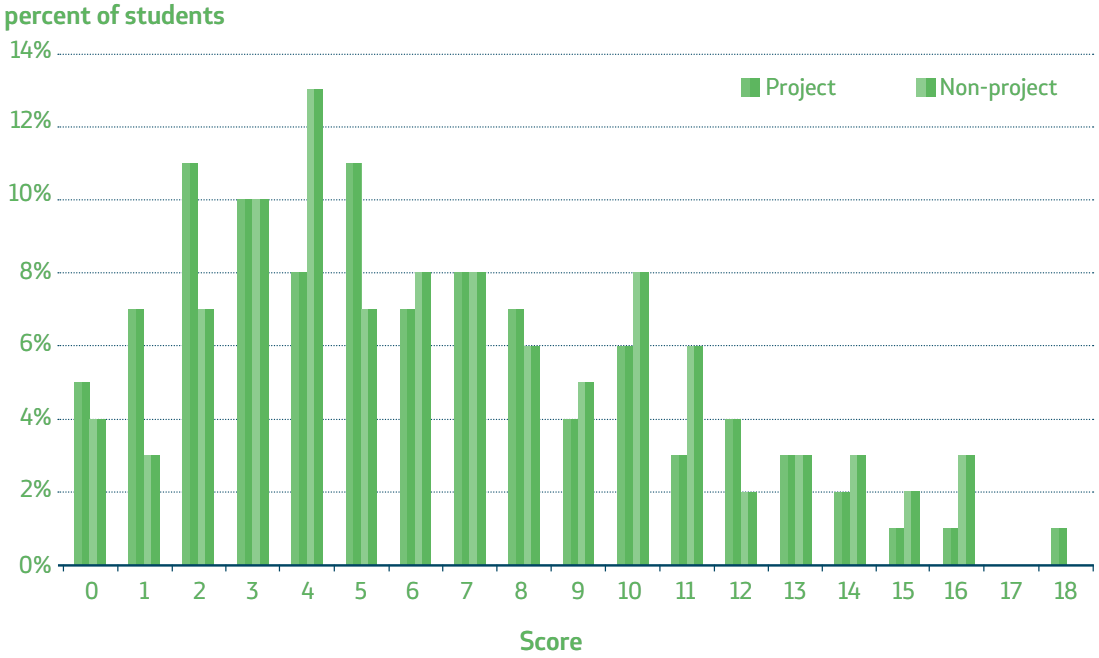


Graph 31. *Results of all students on Operations and properties of operations*

The average score of **students from project schools** on items related to operations and properties of operations is 5.97, i.e. the average percent is 33.17%. The maximum score (18) of students from project schools in this area is achieved by 2 students. The largest is the percent of students (11.04%) who have a score of 2 or a score of 5.

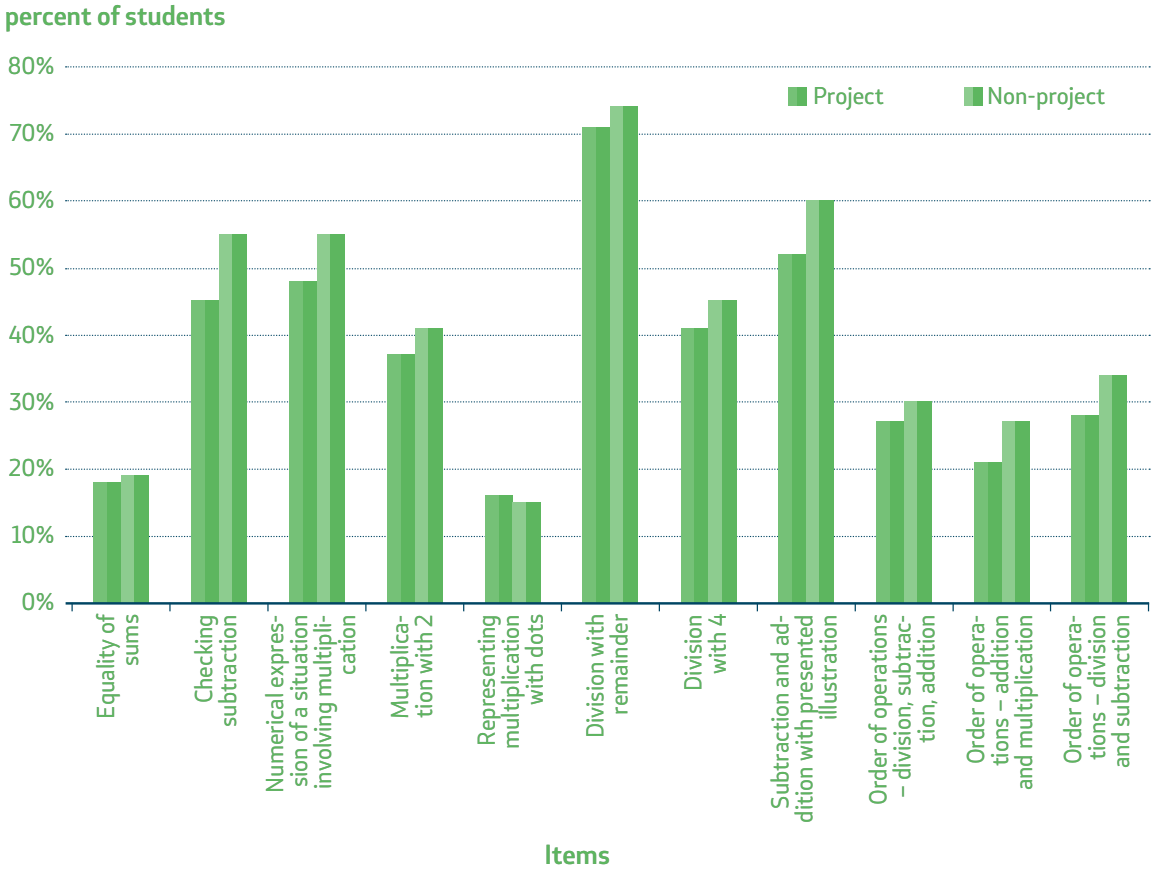
The average score of **students from non-project schools** on the tasks is 6.76, i.e. the average percent is 37.57%. The maximum score (correct responses to all tasks in this area) is achieved by one student only, and out of all tested students 13 did not give correct response to any of the tasks. In non-project schools, the highest is the percentage of students (13.09%) who have a score of 4.

The graph below shows the percent of students from project and non-project schools, according to achieved scores.



Graph 32. *Results of students from project and non-project schools according to items*

The graph below shows the percents of correct responses of students from project and non-project schools, on each of the items given in the test.

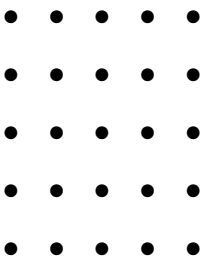


Graph 33. *Achievement of students from project and non-project schools according to tasks*

The least solved item in this area is given below.

Task

Present multiplication of 4•3 on the drawing below..



Correct response and partially correct response was accepted in assessing this item. The table below gives the possible solutions and the percent of students that gave correct, partially correct and incorrect responses.

	Solution	% of resp.
Correct	ON THE DRAWING, the dots are encircled or somehow denoted in 4 rows with 3 dots OR в in 4 rows with 3 dots	10
Partially correct	• 12 dots drawn aside and encircled or not encircled • 12 dots encircled or marked, with no visible grouping where 4•3 could be noticed	10
Not correct	• Groups of 12 dots drawn aside, where 4•3 (for ex. 6 groups with 2; 2 groups with 6), could be noticed • Any of the responses that is not correct	80

Students responses show that:

- Such a way in visual presenting of multiplication (on a spotted paper or on a peg board), is an approach which is rarely or never used in teaching and learning of multiplication;
- Very often teachers insist on learning the multiplication table "by heart", without insisting on understanding the multiplication.

This implies that, many students who wrote the result 12, and even those who didn't do the task, were probably puzzled by the requirement, "to represent multiplication".

This task, in a specific contest, was given to teachers, also - two thirds of them did not give correct response, and among them there were teachers that recorded only $4 \cdot 3 = 12$.

CONCLUSION

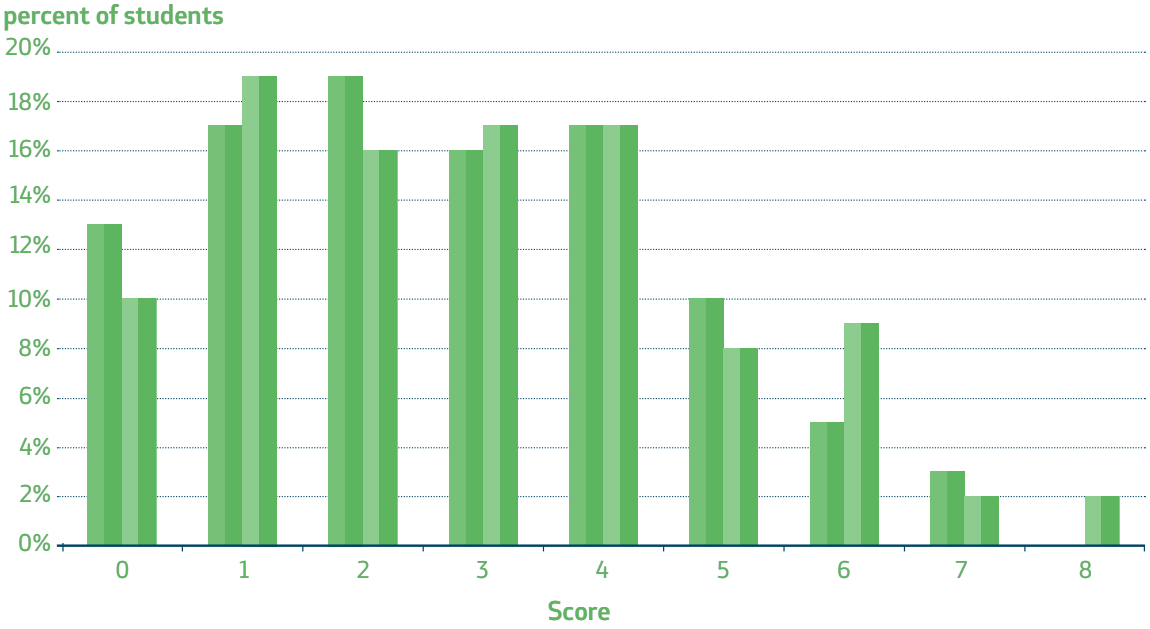
- There is no statistically significant difference in the achievements of students from project and non-project schools.
- The results of tasks in the test which measured the knowledge and skills in operations and properties of operations are lower than the expected results prescribed by the Grade 3 curriculum.

4.1.3. Students achievements on Textual tasks and problems

The average score of **all students** on textual tasks and problem situations is 2.82 (maximum possible is 8), i.e. the average percent of correct answers is 35.26%. The maximum score in this area is achieved ba 6 students (1.01%), and the highest is the percent of students (18.26%) who have a score of 1. Out of the total number (597) tested students 11.39% did not give correct response to any of the items. The achievements at the testhave a normal distrubution which is much more inclined to the left.

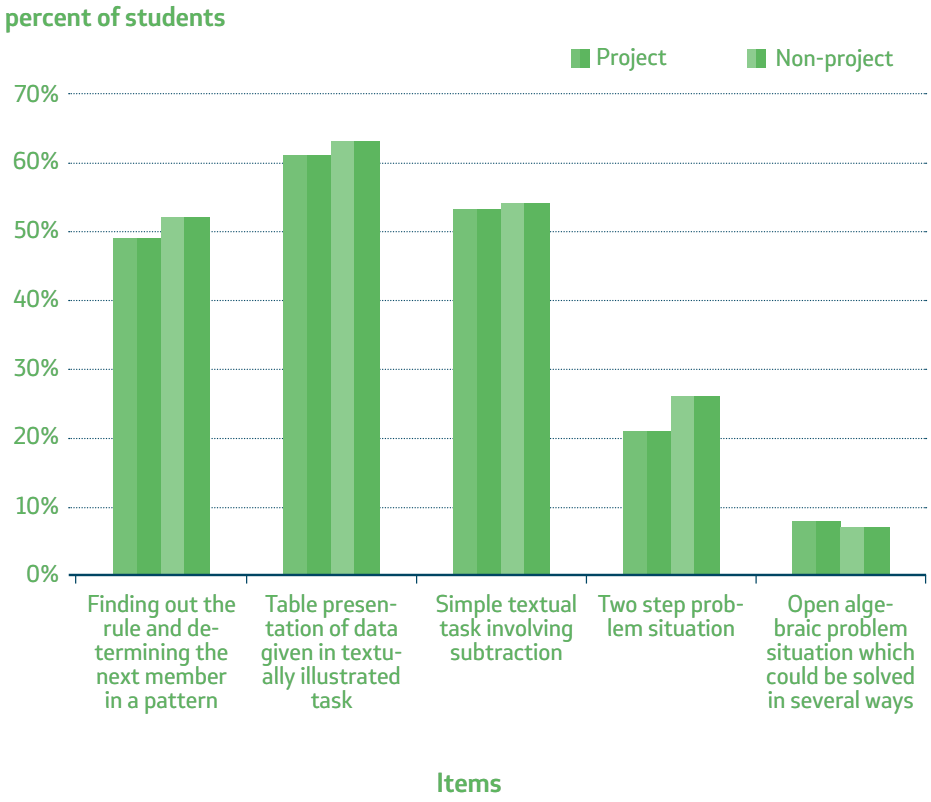
Graph 34. *Results of all teachers on Textual tasks and problems*

- The average score of **students from project schools on** textual tasks and problems is 2.74, i.e. the average score is 35.26%. The maximum score (8) of sudents from project schools in this area is achieved by 1 student only (0.33%). The highest is the percent of students (19.40%) who have a score of 2.
- The average score of **students from non-project scools** on the tasks is 2.90, i.e. the average percent is 36.28%. The maximum score (correct responses to all items in this area) is achieved by 4 students, and out of the total number of tested students 10,07%, did not give correct responses to any of the tasks. In the non-project schools, the largest is the percent of students (19.46%) who have a score of 1.



Graph 35. *Results of students from project and non-project schools according to tasks*

The graph below shows the percents of correct responses of students for each item.



Graph 36. *Achievement of students from project and non-project schools according to items*

The worst solved item in this area is the open problem situation which could be solved in several ways (by drawing, graphically, by table, with guessing and checking, solving it backwards, by equation).

Task

Ivo, a Grade 7 student, sells used colored pencils. Ivo sells 2 colored pencils for 3 denars.

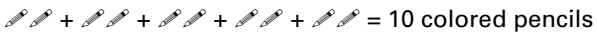
✎✎ = 3 denars

If Ivo has earned 15 denars, how many colored pencils has he sold?

Show how you did it:

Answer: _____ colored pencils.

In marking this item, only correct and partially correct responses were accepted. The table below gives descriptions of the possible solutions and the percent of students that gave correct, partially correct or incorrect rsponses.

<table>
<tr><th></th><th>Solution</th><th>% of resp.</th></tr>
<tr><td>Correct</td><td>
By drawing:
3 + 3 + 3 + 3 + 3 = 15 denars
 = 10 colored pencils
Arithmetical:
15 : 3 = 5 5 • 2 =10
Response: 10 colored pencils
By table:
<table>
<tr><th>denars</th><th>colored pencils</th></tr>
<tr><td>3</td><td>  </td></tr>
<tr><td>6</td><td>  </td></tr>
<tr><td>9</td><td>  </td></tr>
<tr><td>12</td><td>  </td></tr>
<tr><td>15</td><td>  </td></tr>
</table>
Response:10
or
<table>
<tr><th>denars</th><th>colored pencils</th></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>6</td><td>4</td></tr>
<tr><td>9</td><td>6</td></tr>
<tr><td>12</td><td>8</td></tr>
<tr><td>15</td><td>10</td></tr>
</table>
<table>
<tr><th>denars</th><th>colored pencils</th></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>15</td><td>10</td></tr>
</table>
Response: 10 colored pencils
If 2 colored pencils cost 3 denars, one costs a denar and a half.
It means 15 denars, for 10 colored pencils.
</td><td>5</td></tr>
<tr><td>Partially correct</td><td>Any of the mentioned procedures, but the response is 5 colored pencils (2 colored pencils treated as 1).</td><td>6</td></tr>
<tr><td>Incorrect</td><td>Any solution (numerical statement pr procedure) which do not leads to correct or partially correct response.</td><td>89</td></tr>
</table>

Almost all of the students who did the task correctly or partially correctly – did it in an arithmetic way. This implies to the way on which students are taught, to the lack of recommendations for solving the tasks in various ways, as well as to accepting more possible solutions of a given problem. It is obvious that the arithmetic way is most preferred and valued by the teachers.

This task, but in a different context, was given in the test for the teachers – where more than 2/3 of teachers, as a possible way of solving the task, accept only the arithmetic one

CONCLUSION

- There is no statistically significant difference in the achievements of students from project and non-project schools in doing textual tasks and in solving problem situations.
- The results at these tasks in the test are lower than the expected ones, and lower than the results prescribed by the curriculum for Grade 3.

PART IV – CONCLUSIONS AND RECOMMENDATIONS

This part presents the conclusions and the recommendations that could be given on the basis of the findings in the baseline study. They should be used by the managers of the Project in planning the activities for implementation and in the evaluation of the project activities.

CONCLUSIONS

1. The sample of the project schools and that one of the non-project schools do not differ in relation to the relevant characteristics. It enables that they could be easily used in monitoring and evaluation of the effects from project activities.

2. Instruments used to measure particular indicators have satisfactory characteristics, so could be used in subsequent measuring.

3. There are no statistically significant differences in the key indicators between the teachers from the project schools and those from the non-project schools. It enables easy monitoring of the effects on teachers' attitudes and knowledge.

4. There are no statistically significant differences in students' achievements from project and from non-project schools. It enables easy monitoring of the effects of project activities on students' achievements.

5. Teachers have positive attitudes to mathematics and to the teaching of mathematics. It is a good base for introducing new approaches in the teaching of mathematics.

6. Teachers, generally, have attitudes to mathematics and to the teaching mathematics that are in agreement with the Ten Principles of the project *Thinking Mathematics.* It is probably due to their knowledge about the characteristics of a good instruction and learning, which they got as a result of taking part in other projects that promote active methods in teaching.

7. Teachers, responses also show that they accept, to a great extent, the approach in which instruction is stricktly dictated by the curriculum, the insufficiently differentiated approach, the didactically focused instruction and the teaching style, promoting mathematics as a discipline that is mainly a sum of notions, rules, steps, procedures, definitions.

8. Pedagogical knowledge of teachers which would be in accordance with the approach in *Thinking Mathematics* are limited.

9. Mathematics' knowledge and the knowledge for teaching mathematics in accordance with the requirements of the project *Thinking Mathematics* are, also, limited.

10. Teachers' expectations concerning the abilities of their students to do particular mathematical tasks show that they underestimate the abilities of students at a particular age. Similar to this, the majority of teachers consider that students could partly achieve more than that prescribed by curricula.

11. Teachers, in general, consider that curricula enable methods freedom in their instruction. It is a good base for the project activities.

12. Teachers are not sufficiently familiar with the mathematics' curricula for the subsequent cycles in primary education, an especially with the last one.

13. Teachers consider that they cooperate well with their colleagues and have a sense that they could count on support in schools in introducing innovations. According to the statements of school principals and pedagogues /psychologists, grade teachers, in general, cooperate well, but the cooperation with mathematics' subject teachers is insufficient.

14. The managing staff in project schools is willing to support the project activities, though half of them when they were interviewed mentioned that they did not have enough information about the Project.

15. The managing staff, in general, consider that the achievements in mathematics in the grade teaching cycle are good, but almost all of them consider that the achievements could be improved mainly by professional training of teachers.

16. The equipment of classrooms with teaching aids for the project *Thinking Mathematics* is not satisfactory.

RECOMMENDATIONS

1. In-school training is to be carried out through adequately balanced activities that would enable raising teachers' pedagogical and mathematical knowledge.

2. Training should discuss, promote and insist on active approach to teaching (using of multiple differentiated approach and various cooperative techniques, using manipulative aids, tasks related to the environment, tasks that could be solved in many ways or have many solutions, activities through which students themselves should discover the concepts and the rles, etc.)[12]

3. In the course of training, it is necessary to provide access to adequate manipulative aids – so that teachers could experience the need and the usefulness of their application.

4. Support is to be provided to grade teachers (Grades One through Three) by the mathematics teachers in the upper grades, who would help them in raising the level of mathematical knowledge.

5. Subject teachers together with grade teachers, (where there exists such opportunities), should be involved as trainers in the training.

6. UNICEF/BDE, are to organize meetings with school directors, at which they would be informed about the Project, the course of activities and the expected outcomes. The openness for cooperation of the managing staff should be used in providing adequate support to teachers for taking part in training and in applying the new-acquired knowledge.

7. To provide the needed teaching aids (related to the project concepts) for each classroom (a large part of the manipulative teaching aids could be made of cheap materials and by the school itself).

12 Based on the Program for training in *Thinking Mathematics* developed by UNICEF, The Concept for Nine Year Primary Education and the Methods recommendations in mathematics' curricula for grades one through three.

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APPENDICES

APPENDIX 1:

SAMPLE

Sample of schools

- **15 project schools**

Method of selection:

From the list of 35 project schools, 15 were selected according to the following criteria:

- Regional coverage (to have schools from different regions that were involved in the project)
- Language of instruction:
- 9 with Macedonian language of instruction, 3 with Macedonian and Albanian language of instruction and 3 with Albanian language of instruction
- Urban/rural
- **5 from Skopje, 6 from other towns and 3 rural**
- **15 parallel non-project schools with similar characteristics as the project ones concerning geographical coverage, the location, the language of instruction, students' social background.**

The selected schools are shown in the following table

No.	Project			Non-project	
	School	Location	Language of instruction	Location	School
1	Lazo Angelovski	Skopje	Mac.	Skopje	Gjorgija Pulevski
2	J. H. Pestaloci	Skopje	Mac.	Skopje	Kole Nedelkovski
3	J. A. Komenski	Skopje	Mac.	Skopje	Vera Ciriviri Trena
4	25.Maj	Skopje	Alb.	Skopje	7-mi Mart
5	Aco Sopov	Skopje	Mac..	Skopje	Zivko Brajkovski
6	Alija Avdovik	Batinci	Alb.	Glumovo	Sami Fraseri
7	Hristo Uzunov	Ohrid	Mac.	Ohrid	Gligor Prlicev
8	Bratstvo Edinstvo	Ohrid	Mac. and Alb.	Struga	Brakja Miladinovci
9	Kiril Pejcinovic	Tearce	Mac. and Alb.	Vratnica	Simce Nastevski
10	Kiril i Metodi	Тетово	Mac. and Alb.	Tetovo	Lirija
11	Dituria	Lipkovo	Alb.	Slupcane	Faik Konica
12	11 Oktomvri	Kumanovo	Mac.	Kumanovo	Brakja Miladinovci
13	Kiril i Metodij	Kocani	Mac.	Kocani	Nikola Karev
14	Sando Masev	Strumica	Mac..	Strumica	Hristo Uzunov
15	D A. Gaberot	Kavadarci	Mac.	Kavadarci	Tode Hadji Tefov

Sample of teachers:

- Random sample of 10 grade teachers from each school[13]

Sample of school principles and pedagogues/psychologists:

- All (15) school principals from project schools and one pedagogue or psychologist

Sample of students:

- Random sample of 20 Grade 4 (new) students from each of the selected schools[14].

13 The method of selection is explained in the *Manual for the researcher* (Appendix 2)
14 The way of selection is explained in the *Manual for the researcher*

APPENDIX 2.

PROJECT: THINKING MATHEMATICS
IN GRADES 1, 2 AND 3

GUIDELINES TO RESEARCHER

The researcher for each of the schools is expected to carry out the following activities needed for developing the Baseline study prior to starting the *Project*:

1. To collect data about the school and about the students
2. To administer the testing of Grade 4 students (new)
3. To administer the Questionnaire for the teachers
4. To interview the school principal and the school pedagogue/psychologist
5. To write a report about the conducted activities
6. To submit all materials to MCEC

Each one of the mentioned activities is explained below in details.

Skopje, November 2009

МАКЕДОНСКИ
ЦЕНТАР ЗА
ГРАЃАНСКО
ОБРАЗОВАНИЕ

The study is conducted by the
Macedonian Civic Education Centre

MANAGEMENT OF THE INVESTIGATION

- Call the school at least two days prior to your visit to the school and explain them the purpose of your visit related to collecting *a basline data prior to starting the training within the UNICEF Project Thinking Mathematics*
- Explain them that the data will be used by UNICEF in planning the Project activities
- Explain them what activities on the day of your visit to the school, you are expected to carry out (testing, conducting survey, interview)
- Explain them how to select the sample of students for the testing (see part 2. Testing students)
- Explain them how to select the sample of teachers to give responses to the questionnaire (see part 2. Giving responses to the questionnaire)
- Explain them that the selected teachers from the grade teaching cycle are expected **to be present** and to fill in the Questionnaire for teachers
- Agree with them about the time schedule for each one of the activities

You could send them the form for school's data in advance (provided the school has access to internet).

1. COLLECTING DATA ABOUT THE SCHOOL

- Data about the school should be entered into the form **School Data**.
- You could collect the data during your visit to the school or send the form to the school prior to your visit, so that the school could fill it in, and in exceptional cases to send it to you back after your visit.

2. TESTING OF GRADE 4 STUDENTS (NEW)

- It is necessary to test 20 Grade 4 students (new).
- **Method of selection:** The total number of Grade 4 students (new) having classes in building of the central primary school, is divided by (the decimal number is encircled as a whole number). The obtained number is: **N – step of choice**. A common list of Grade 4 students (new) from all classes is made. Starting from the number 2, you choose every **N-th** student, until you have chosen 20 and then you go on the circle from the beginning). For ex. There are 90 students. 90:20=4.5. Encircled is number 5. Starting with number 2 every 5th student is selected: 2,7,12,17,22.... until you have selected 20 students.

Provided that the school has two languages of instruction, 10 students are selected per language of instruction.

- The selection of students is done one day prior to the testing.
- **Venue:** The selected students are assembled in one classroom.

Provided that the school has two languages of instruction sudents are tested separately, 10 per group in Macedonian and in Albanian language of instruction.

- **Time schedule of testing:** It is best that the testing be administered during the second class-hour.
- **Instruction for testing:** You explain to students that they would do a test in mathematics and that it is not intended for them to get marks, but to see how children from different schools have learnt mathematics. You explain them how to do the test, to record the responses, and you should provide that they have understood all that. Then you could begin with the testing. During the testing you could give explanation only about the way of giving the responses (recording of responses).
- **Timeline of the testing:** Testing lasts 40 minutes. You could give instructions to students, that have completed doing the test earlier, to check the solutions. In case, when within 40 minutes more then half of the students haven't completed doing the test, extend the time for additional 5 – 10 minutes (to allow half of the students to complete doing the test). In the report about the testing, you should record that you have extended the timeline for testing.
- **Returning the tests:** you should return all the tests, no matter they are filled in or not.

3. ADMINISTERING THE QUESTIONNAIRE FOR TEACHERS

- The questionnaire for teachers should be filled in by 10 sellected teachers from the grade teaching cycle working in the Central primary school
- **Method of selection:** The total number of clases from Grades 1 – 4, studying in the building of the Central primary school, is divided by 10 (the decimal number is encircled as a whole number). The obtained number is: **N – step of selection**.
- A common list of all classes in Grades I through IV is made in the following way. I_1 (a); I_2 (b)…. II_1 (a); II_2 (b)…. III_1 (a); III_2 (b)…. IV_1 (a); IV_2 (b)….So, the ordered ones are denoted by numbers 1, 2, 3....
- Starting from number 2, you select every N-the class, until you have selected 10 classess (if you come to the end of the list before you have selected 10 you repeat the circle from the beginning). For ex. In grades 1 through 4 there are 23 classes. 23 : 10 = 2,3. You encircle the number 3. Starting from number 2 you select every 2^{nd} class: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. Teachers that teach in the selected classes are chosen to fill in the questionaire. In case that a whole day teaching class is selected then you choose the teachers who teach mathematics.

Provided that the school has two languages of instruction, 5 teachers are selected per language of instruction.

- **Venue:** Assemble the selected teachers in one room (teachers' room, classroom), where they would not be disturbed for about 70 minutes.
- **Time schedule:** The best time for filling in the questionnaire should be agrreed upon, during the period after the classes.
- **Instruction:** Explain to teachers that the purpose of giving responses to the questionnaire is: to provide a baseline prior to starting the training in the UNICEF Project. Explain to them that the responses are anonymous. Ask them to work independently, because we consider their individual responses as important for us. Provide that they work independently.
- **Timeline:** About 60 minutes are needed to fill in the questionnaire. You could organize to do it in two parts with a coffee break of 5 – 10 minutes. In that case ask them to write a code on their questionnaire and then to continue filling in his/her questionnaire.
- **Returning back the questionnaires:** you should return all the questionnaires that you have received, no matter the are filled in or not.
- **Времетраење:** За пополнување на прашалникот се потребни околу 60 минути. Може да го организирате да се пополнува во два дела со пауза од 5-10 мин. Во тој случај обезбедете секој да напише шифра на својот прашалник и да продолжи да одговара во својот прашалник.
- **Враќање на прашалниците:** треба да ги вратите сите прашалници што сте ги добиле, без оглед дали се пополнети или не се.

4. CONDUCTING THE INTERVIEW WITH THE SCHOOL PRINCIPLE AND THE PSYCHOLOGIST/PEDAGOGUE

- It is necessary to interview the school principle and one pedagogue /psychologist.
- During the interview no one else should be present, exept the person being interviewed
- **Time schedule:** Agree with the person to be interviewed the time that suits him/her and you (taking into consideration the other activities for collecting the data).
- **Instruction:** Explain them the purpose of the interview: *to provide baseline data prior to starting the training in the UNICEF Project.* Provide a spontaneous and not too much formal discussion. It is necessary to provide responses to all questions. Record the responses in the Reminder for the interview.
- **Timeline:** You would need at most 30 minutes for the interview.

REPORT

- In the report, you should record briefly all conducted activities related to the survey
  - Previous preparations and arrangements
  - Administering the testing
  - Administering the questionnaire
  - The interviews
  - Data about the school
  - Estimate of the researcher concerning the successful implementation, limitations related to the collected data,etc..
- Together with the Report you should submit all the materials.

APPENDIX 3:

INSTRUMENTS AND PSYCHOMETRIC CHARACTERISTICS OF INSTRUMENTS

Scale of attitudes in learning mathematics and in teaching mathematics based on the Ten Principles

Content

The scale of attitudes to learning mathematics and to teaching mathematics consist of 30 statements related to the understanding how children learn mathematics and to the acceptance of different approaches and situations related to teaching of mathematics. The statements are related to the Ten Principles of *Thinking Mathematics.*

Structure of the scale is as following:

Content:

- 15 statements relate to the understanding how children learn mathematics;
- 15 statements describe approaches in teaching mathematics;
- Each one of the Ten Principles is presented with 2 – 4 statements.

Formulation:

- 13 statements are formulated positively;
- 17 statements are formulated negatively.

The instruction was: **In relation to mathematics, various people have different experience and opinion. Below are given statements in relation to mathemetics.**

There are no correct and incorrect responses. Your responses should as much as possible express YOUR opinion.

According to your experience, on the scale from 1 (I do not agree at all) to 5 (I fuly agree) encircle the number which best denotes the degree in which you agree with Each One of the following statements.

Process of construction

Due to the fact that we were not familiar with the scale used to measure attitudes (opinions, beliefs) related to the instruction based on the Ten Principles of *Thinking Mathematics,* for the needs of this survey we developed such a scale. Other instruments that examine teaching styles and the attitudes (beliefs) related to learning mathematics, were also consulted.

First, 70 statements related to the instruction adjusted to the Ten Principles, were formulated. They were given to 4 participants at the training for *Thinking Mathematics* and they were asked to relate the statements with the Ten Principles. On the basis of their responses, 30 statements were selected, to which at least three teachers agreed upon. In such a way construct validity was provided. Care was taken that each principle should be related to at least 2 statements.

In such a way, the selected statements are ordered in the scale which was given during the investigation, so that statements for the same principle should not stand one by the other, and the positively and negatively formulated statements not to be grouped together.

Psychometric characteristics

After giving the Scale to 299 teachers from project and non-project schools the following psychometric characteristics were stated:

- The item test correlation ranged from 0.40 to 0.55 with 11 statements, from 0.30 to 0.39 with 17 statements, and with 2 statements it was 0.25. It points out to a satisfactory item test correlation of the statements.

Reliability (Cronbach alpha coefficient) is 0.86, which is a satisfactory reliability.

Scale of attitudes to mathematics

Content and process of construction

The scale of attitudes to mathematics contains 23 statements related to mathematics and to the teaching of mathematics. They refer to their opinion concerning:

- the nature and relevance of mathematics
- learning mathematics (experience from learning mathematics)
- personal competence in mathematics (preferences for work on mathematical problems and the selfconfidence related to the teaching of mathematics)[15]

Part of the statements (12) are adapted from Minnesota Mathematics Attitude Inventory[16] and the scale used by Relich, Way and Martin (1994)[17], and the remaining (11) are developed for the needs of this survey.

15 Mantecón J. D., P. Andrews and P. Op 't Eynde, Refining The Mathematics-Related Beliefs Questionnaire (MRBQ), http://ermeweb.free.fr/CERME%205/WG2/2_Diego-Mantecon.pdf

16 Minnesota Mathematics Attitude Inventory, University of Minnesota www.crosspulseconsultants.com/MMAI.pdf

17 Според White A., L. White, J. Way, B. Perry, B. Southwell, (2006), Mathematical Attitudes, Beliefs and Achievement in Primary Pre-service Mathematics Teacher Education, *Mathematics Teacher Education and Development,* 2005/2006, Vol. 7, 33–52

14 of the statements are formulated positively, and 9 negatively.

The instruction was: **Related to mathematics various people have different experience and opinion. Below are given statements by teachers related to mathematics.**

The are no correct or incorrect responses. Your responses should as much as possible express YOUR opinion.

According to your experience, on scale from 1 (I do not agree at all) to 5 (I fuly agree) encircle the number which best denotes the degree with which you agree on EACH ONE of the mentioned statements.

Psychometric characteristics

After giving the Scale to 299 teachers from project and non-project schools the following psychometric characteristics were stated:

- the item test correlation ranged from 0.40 до 0.69, which points out to their good item - test correlation
- Reliability (Cronbach alpha coefficient) is 0.81, which is a satisfactory reliability.

Test in pedagogical knowledge

Content and process of construction

The test in pedagogical knowledge contains 11 vignettes (teaching situation). In each one of them it is required to choose the adequate or the best teacher's reaction. Hypothetic situations are situations in which teacher could react adequately to the approach promoted by *Thinking Mathematics*, or inadequately (traditional).

Teaching situations were developed for the needs of this survey, on the basis of the training materials in *Thinking Mathematics.*

Psychometric characteristics

The description of the content of the teaching situations and their discriminative value (– the item - test corelation) is given in the table below.

The reliability is 0.55. The insufficient reliability is due to the small number of items. Provided that this test would have 40 items items with similar characteristics, the estimated reliability of the test would be 0.78.

The item - test correlation is given below. The table gives the number of items for each one of the categories: *very good item, good item, not satisfactory item,* according to item test correlation.

Category	Item test correlation	Number of items in the test
Very good	41 – 53	8
Good	31 – 40	2
Satisfactory	21 – 30	4

Test in mathematics' knowledge of teachers

Content and process of construction

The test in mathematics' knowledge of teachers contains 15 items, out of which 8 are clusters with 3 or 4 items. The tasks cover knowledge and skills in the following areas:

- The concept of number – 9 items;
- Operations and properties of operations – 19 items and
- Problem situations – 6 items.

The items in the test for teachers, from the aspect of content, do not exeed the curricula for mathematics in primary education, i.e. the expected results from students at the end of Grade 6.

Part of the tasks (5) are adapted from Learning Mathematics for Teaching (MKT), Mathematics Released Items (2008)[18] and (2) from The Effects of Different Undergraduate Mathematics Courses on the Content Knowledge and Attitude towards Mathematics of Pre-service Elementary Teachers (2007)[19]. The remaining (8) are developed for this survey.

Psychometric characteristics

The test reliability is 0.74 and is due to the smaller number of items. Provided that this test would have had 40 items with similar characteristics, the estimated reliability of the test would be 0.77.

The table below shows the number of items for each of the categories: *very good item, good item, satisfactory, and bad item,* according to the item according to the item test correlation.

Category	Item test correlation	Number of items in the test
Very good	41 – 65	10
Good	31 – 40	13
Satisfactory	21 – 30	7
Bad	12 – 20	4

18 Learning Mathematics for Teaching Project (2008), University of Michigan, www.sitemaker.umich.edu

19 IUMPST: The Journal ,Vol 1 (Content Knowladge, July 2007 [www.k-12prep.math.ttu.edu]

Test in mathematics knowledge of students

Content and process of construction

The test used to measure students' knowledge has 21 item from the areas covered by the Project,i.e.:

- The concept of number;
- Operations (addition, substraction, multiplication and division) and properties of operations; and
- Solving textual tasks and problems containing operations, models and work with data.

9 items with multiple choice answers were used, 6 with short answer and 4 open ended, which required complete procedure and solution. The items in this test were specially developed for this survey.

Prior to developing the final version of the test, 23 tasks (including the 19 tasks in the final test) were administered on 20 Grade 4 students (the new), in one of the project schools which is not in the sample for this study. Time to do the test by students was measured also. On the basis of students' responses, the tasks were improved, and 4 were removed in order to give students opportunity to the test in no more than 40 minutes. The final version of the test after its translation into Albanian language, was checked again from the aspect of adequacy of the translation, in accordance with students' age and understanding.

Psychometric characteristics

The test was administered on 597 students (from the sample of project and non-project schools). On the basis of the outcomes, the reliability of the administered test is 0.79, and is due to the smaller number of items. Provided that this test would have 40 items with similar characteristics, the estimated reliability of the test would amount 0.88.

The table below, shows the numbers of items for each of the categories: *very good item, good item, satisfactory and poor item* due to the item test correlation.

Category	Item test correlation	Number of items in the test
Very good	41 – 65	14
Good	31 – 40	6
Satisfactory	2 1 – 30	1
Poor	< 20	0

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